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From Relative Entropy to the Semiclassical Einstein Equations

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Motivation

Jacobson showed that the Einstein equations

$$R_{\mu\nu} - \tfrac{R}{2} g_{\mu\nu} + \Lambda\, g_{\mu\nu} = 8\pi\, T_{\mu\nu}$$

can be derived from the thermodynamic relation $\delta Q = T\,\delta S$ and the Bekenstein-Hawking entropy-area formula $S_{BH} = \frac{A}{4}$, applied to local Rindler horizons (Jacobson, 1995).

- The argument invokes the Unruh and Hawking effects, which cannot be accounted for in classical general relativity.
- A complete derivation requires incorporating a quantum field theoretical matter model.

Previous attempts to bridge this gap employ the von Neumann entanglement entropy. However, this quantity is mathematically ill-defined in QFT, since the von Neumann algebras describing QFTs are typically of type III.

This work aims to resolve these issues by using the *relative entropy* (Araki, 1976 & Uhlmann, 1977), which quantifies the distinguishability of two quantum states and remains mathematically well-defined for type III von Neumann algebras.

Local Spacetime Geometry

By the equivalence principle, the spacetime metric $g_{\mu\nu}$ in a sufficiently small spacetime region $\mathcal{U}$ is well-approximated by a local inertial frame, i.e.

$$g_{\mu\nu}\big|_{\mathcal{U}} \approx \eta_{\mu\nu}.$$

The generator ξ^μ of Lorentz boosts therefore constitutes an approximate Killing vector field, so that the associated local Rindler horizon is an approximate bifurcate Killing horizon (Jacobson, 1995).

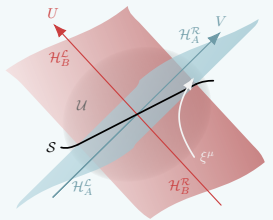


Figure 1: Sketch of a local Rindler horizon, consisting of the null hypersurfaces $\mathcal{H}_A$ and $\mathcal{H}_B$ which intersect at the spacelike two-surface $\mathcal{S}$. $\mathcal{H}_A$ and $\mathcal{H}_B$ are affinely parametrized by V and U, respectively. Accordingly, the region $\mathcal{U}$ is separated into two wedge-shaped regions, decomposing $\mathcal{H}_A, \mathcal{H}_B$ into $\mathcal{H}_A^L, \mathcal{H}_A^R$ and $\mathcal{H}_B^L, \mathcal{H}_B^R$. The white arrow depicts the local approximate boost Killing vector field ξ^μ.

Quantum Fields on the Local Horizon

We consider a quantized, free, real scalar field Φ on $\mathcal{U}$, described by the von Neumann algebra $\mathcal{N}$.

- The restriction of the local vacuum state ω_0 on $\mathcal{N}\big|_{\mathcal{U}}$ to the horizon $\mathcal{H}_B$ is for $f, h \in C_0^\infty(\mathcal{H}_B)$ given by the two-point function (Kay & Wald, 1991)

$$\Lambda(f,h) = -\frac{1}{\pi}\int \frac{f(U,p_S)\,h(U',p_S)}{(U-U'-i0^+)^2}\,dU\,dU'\,d\mathrm{vol}_S.$$

The corresponding von Neumann algebra $\mathcal{N}_B^R$ of observables localized on $\mathcal{H}_B^R$ is constructed as:

- $C_0^\infty(\mathcal{H}_B^R)$ is equipped with the symplectic form

$$\sigma(f,h) = \int_{\mathcal{H}_B^R} \big((\partial_U f)\,h - (\partial_U h)\,f\big)\,dU d\mathrm{vol}_S.$$

- There exists a unique associated Weyl algebra $\mathscr{W}_B^R$ to the symplectic space $\big(C_0^\infty(\mathcal{H}_B^R), \sigma\big)$.
- For each $f \in C_0^\infty(\mathcal{H}_B^R)$ and $W(f) \in \mathscr{W}_B^R$, the state ω_0 is given by $\omega_0\big(W(f)\big) = e^{-\frac{1}{2}\Lambda(f,f)}$.
- ω_0 on $\mathscr{W}_B^R$ satisfies the KMS condition at inverse temperature $\beta = \frac{2\pi}{\kappa}$ w.r.t. the flow generated by ξ^a (Kay & Wald, 1991; Summers & Verch, 1996).

The von Neumann algebra $\mathcal{N}_B^R$ on $\mathcal{H}_B^R$ is then obtained as the double commutant $\mathcal{N}_B^R = \pi_0(\mathscr{W}_B^R)''$ of the GNS representation π_0 of $\mathscr{W}_B^R$.

- The GNS vector Ω_0 is cyclic and separating for the horizon algebra $\mathcal{N}_B^R$.
- Hence, the modular operator Δ_R for $(\mathcal{N}_B^R, \Omega_0)$ acts geometrically as affine dilations along $\mathcal{H}_B^R$, i.e. $\Delta_R^{it} = \mathfrak{D}_{2\pi t}$ (Summers & Verch, 1996).

Relative Entropy of Coherent States

The relative entropy between ω_0 and a coherent excitation $\omega_f\big(W(h)\big) = \omega_0\big(W(f)^* W(h) W(f)\big)$ can be computed by the formula (Longo, 2019)

$$S^{\mathrm{rel}}(\omega_0\|\omega_f) = i\,\tfrac{d}{dt}\big|_{t=0}\,\langle\Omega_0 \mid \pi_0(W(f)^*)\Delta_R^{it}\,\pi_0(W(f))\,\Omega_0\rangle,$$

which explicitly gives (Kurpicz, Pinamonti & Verch, 2021)

$$S^{\mathrm{rel}}(\omega_0\|\omega_f) = -2\pi \int_{\mathcal{H}_B^R} U(\partial_U f)^2 dU d\mathrm{vol}_S.$$

This result can be re-expressed in terms of the expectation value $\langle :T_{\mu\nu}: \rangle_{\omega_f}$ of the regularized energy-momentum tensor of Φ:

- The expectation value of the energy density in the state ω_f, along the flow of ξ^μ, is given by

$$\langle :T_{\mu\nu}: \rangle_{\omega_f}\,\xi^\mu\xi^\nu = (\partial_U f)^2.$$

- Substituting this relation into the formula for the relative entropy gives

$$S^{\mathrm{rel}}(\omega_0\|\omega_f) = -2\pi \int_{\mathcal{H}_B^R} U \langle :T_{\mu\nu}: \rangle_{\omega_f}\,\xi^\mu\xi^\nu dU d\mathrm{vol}_S.$$

- This expression is equal to the semiclassical energy flux across the local Rindler horizon (Kay & Wald, 1991).

Recovering the Semiclassical Einstein Equations

Assuming that the influx of quantum matter through the local Rindler horizon induces a variation $\delta\mathcal{A}$ of the horizon area, it follows that

$$S^{\mathrm{rel}}(\omega_0\|\omega_f) = \alpha\,\delta\mathcal{A}\,.$$

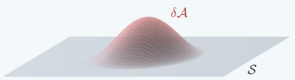


Figure 2: Visualization of the local area variation $\delta\mathcal{A}$ of the spacelike horizon cross-section $\mathcal{S}$, induced by the semiclassical energy flux $\delta Q = -\int_{\mathcal{H}_B^R} U \langle :T_{\mu\nu}: \rangle_{\omega_f} \xi^\mu\xi^\nu dU d\mathrm{vol}_S$ for some compactly supported function $f \in C_0^\infty(\mathcal{H}_B^R)$.

Geometrically, $\delta\mathcal{A}$ can be expressed in terms of the expansion scalar θ of null geodesics along $\mathcal{H}_B$ with tangent vector ξ^μ (Poisson, 2004), i.e.

$$\delta A = \int_{\mathcal{H}_B^R} \theta\, dU d\mathrm{vol}_S.$$

- θ is governed by the Raychaudhuri equation

$$\tfrac{d\theta}{dU} = -\tfrac{\theta^2}{2} - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}\,\xi^\mu\xi^\nu.$$

- On any approximate Killing horizon, we have $\theta^2 \approx 0$ and $\sigma_{\mu\nu}\sigma^{\mu\nu} \approx 0 \approx \omega_{\mu\nu}\omega^{\mu\nu}$ (Poisson, 2004), which yields the geometric relation

$$\delta\mathcal{A} = -\int_{\mathcal{H}_B^R} U R_{ab}\,\xi^a\xi^b\, dU d\mathrm{vol}_S.$$

Equating both expressions for $\delta\mathcal{A}$ and invoking local energy-momentum conservation finally gives

$$R_{\mu\nu} - \tfrac{R}{2} g_{\mu\nu} + \Lambda\, g_{\mu\nu} = \tfrac{2\pi}{\alpha}\,\langle :T_{\mu\nu}: \rangle_{\omega_f}\,.$$

Conversely, assuming that the semiclassical Einstein equations hold automatically fixes the proportionality constant $\alpha = \frac{1}{4}$ and yields

$$S^{\mathrm{rel}}(\omega_0\|\omega_f) = \tfrac{\delta\mathcal{A}}{4}\,.$$

Conclusions

Local spacetime curvature can be interpreted to arise from the distinguishability of a given quantum state from the vacuum.

- If there is no genuine excitation, i.e. $f = 0$, ω_0 and ω_f coincide, so that $\delta\mathcal{A} = 0$ and the local spacetime geometry remains exactly flat.
- The derivation is a weak-curvature approximation, relying on the validity of $\mathcal{U}$ being a local inertial frame with a local Rindler horizon.
- Extensions to strong-curvature regimes and noncoherent states are expected to be possible, but mathematically much more involved.

Link to Paper

Sorkin's paradox and bounds on relative entropy

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based on a joint work with Christoph Minz and Ko Sanders (Leibniz Universität Hannover)

General ideas

Computing **relative entropy** in QFT requires handling **relative modular Hamiltonians**, which have **nice general properties** but are **very difficult to compute explicitly**.

Through locality properties of **general algebraic QFTs** , we can compare

- relative modular Hamiltonians between two suitable states on some region;
- the modular Hamiltonian of a reference state on a larger or smaller region.

When this holds, there exist **non signalling unitaries** in the sense of **Sorkin's paradox**.

Question: can this be used to quantify information between different regions?

preprint 2605.27198
arxiv.org/abs/2605.27198

Relative modular theory in QFT

Given a von Neumann algebra $\mathcal{R}$ on a Hilbert space $\mathcal{H}$, we call Ω a **standard vector** for $\mathcal{R}$ iff it is both

- cyclic, $\overline{\mathcal{R}\Omega} = \mathcal{H}$
- separating, $a\Omega = 0 \Longrightarrow a = 0$, $a \in \mathcal{R}$

Given two standard unit vectors $\Omega, \tilde{\Omega} \in \mathcal{H}$ for $\mathcal{R}$, $a \in \mathcal{R}$,

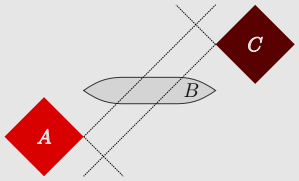
- the **relative Tomita operator** is $S_{\tilde{\Omega},\Omega} a\Omega = a^* \tilde{\Omega}$

Its polar decomposition is $S_{\tilde{\Omega},\Omega} = J_{\tilde{\Omega},\Omega} \Delta_{\tilde{\Omega},\Omega}^{1/2}$, with

- **relative modular operator** $\Delta_{\tilde{\Omega},\Omega} > 0$
- **relative modular conjugation** $J_{\tilde{\Omega},\Omega}$, $J^*_{\tilde{\Omega},\Omega} = J^{-1}_{\tilde{\Omega},\Omega}$
- **relative modular Hamiltonian** $K_{\tilde{\Omega},\Omega} = -\log\left(\Delta_{\tilde{\Omega},\Omega}\right)$
- **relative entropy** $H(\Omega, \tilde{\Omega}) = \langle \Omega, K_{\tilde{\Omega},\Omega}\Omega \rangle$

Sorkin's paradox

We have Alice (A), Bob (B) and Charlie (C):



with algebras $\mathcal{R}_A$, $\mathcal{R}_B$ and $\mathcal{R}_C$ acting on $\mathcal{H}$.

Alice and Charlie are **spacelike separated**, $[\mathcal{R}_A, \mathcal{R}_C] = 0$, an **impossible measurement** in B violates this causality.

A unitary w on $\mathcal{H}$ will be **non-signalling [1,** Sec. III**]**

from A to C iff $w\mathcal{R}_A w^* \subset \mathcal{R}'_C$
from C to A iff $w\mathcal{R}_C w^* \subset \mathcal{R}'_A$

Estimates for modular data

Let $V_1 \subset V_2$ open regions with algebras $\mathcal{R}_1 \subset \mathcal{R}_2$ (isotony)

Lemma. *Let $\mathcal{R}_1 \subset \mathcal{R}_2$ on $\mathcal{H}$ and $\Omega, \tilde{\Omega}$ standard for both:*

$$K^{(1)}_{\tilde{\Omega},\Omega} \leq K^{(2)}_{\tilde{\Omega},\Omega} \text{ and so } H^{(1)}(\Omega, \tilde{\Omega}) \leq H^{(2)}(\Omega, \tilde{\Omega})$$

Theorem. *Now let $u \in \mathcal{R}_2$ and $u' \in \mathcal{R}'_1$ be unitaries:*

1. *If both $u\Omega$ and Ω are standard for both $\mathcal{R}_i$,*

$$K^{(1)}_{u'u\Omega,\Omega} \leq u K^{(2)}_{\Omega} u^*$$

2. *If both $u'\Omega$ and Ω are standard for both $\mathcal{R}_i$,*

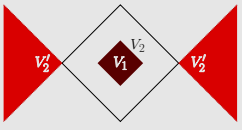
$$K^{(2)}_{uu'\Omega,\Omega} \geq u K^{(1)}_{\Omega} u^*$$

and these directly apply to relative entropy.

The connection

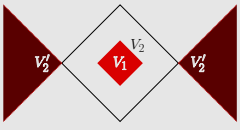
A unitary $w = u'u$ as in the Theorem is **non-signalling**

from A to C (point 1.) if



(consider $A = V_2'$, $C = V_1$)

from C to A (point 2.) if



(consider $A = V_1$, $C = V_2'$)

We can indeed check that respectively

$$\begin{matrix} w\mathcal{R}_A w^* \\ w\mathcal{R}_C w^* \end{matrix} = w\mathcal{R}'_2 w^* = u'u\mathcal{R}'_2 u^* u'^* = u'\mathcal{R}'_2 u'^* \subset \mathcal{R}'_1 = \begin{matrix} \mathcal{R}'_C \\ \mathcal{R}'_A \end{matrix}$$

[1] L. Borsten, I. Jubb and G. Kells. "Impossible measurements revisited". In: *Physical Review D* 104.2 (2021)

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Foundations of Relational Quantum Field Theory: Scalars

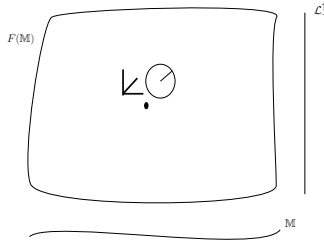
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Abstract

We develop foundations for a relational approach to quantum field theory (RQFT) based on the operational quantum reference frames (QRFs) framework considered in a relativistic setting. Unlike other efforts in combining QFT with QRFs, we use the latter to provide novel mathematical and conceptual foundations for the former. We focus on scalar fields in Minkowski spacetime and discuss the emergence of relational local observables and fields from the consideration of Poincaré-covariant frame observables defined over the space of inertial reference frames. We recover a relational notion of Poincaré covariance, with transformations on the system directly linked to the state preparations of the QRF. We introduce and analyse various causality conditions. The theory makes direct contact with established foundational approaches to QFT: the frame smearing functions describing the QRF's localisation uncertainty plays the role of the Wightmanian test functions, and the algebras generated by relational local observables suitably extend the core axioms of Algebraic QFT.

Relativistic Quantum Reference Frames

Start with a classical inertial reference frame, which can be thought of as a clock and rods, sharply localised in spacetime and with a definite Lorentz orientation. Such a tetrad lives in the (orthonormal) frame bundle over Minkowski $F(\mathbb{M})$.



Suppose you want to describe a system in state $\rho = \rho^{(X)} \in \mathscr{D}(\mathcal{H}_\mathcal{S})$ with system Hilbert space $\mathcal{H}_\mathcal{S}$, relative to your inertial reference frame $X \in F(\mathbb{M})$. The state relative to another inertial reference frame $X' = (x, \lambda) \cdot X$ is $\rho^{(X')} = U_\mathcal{S}(x,\lambda)^\dagger \rho U_\mathcal{S}(x,\lambda)$.

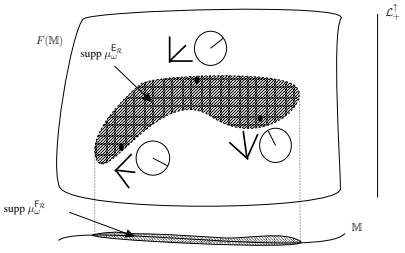
Perhaps you have some uncertainty regarding your inertial reference frame: you assign a probability distribution $\mu \in \mathrm{Prob}(F(\mathbb{M}))$ so that the proper mixed state that you should assign to describe experiments is

$$\rho^{(\mu)} = \int_{F(\mathbb{M})} \rho^{(X)} d\mu(X) = \int_{\mathcal{P}^\uparrow_+} U_\mathcal{S}(x,\lambda)^\dagger \rho U_\mathcal{S}(x,\lambda) d\mu(x,\lambda)$$

or, in a Heisenberg-like picture with fixed states, the observables now look like

$$\phi^{(\mu)} = \int_{\mathcal{P}^\uparrow_+} U_\mathcal{S}(x,\lambda) \phi U_\mathcal{S}(x,\lambda)^\dagger d\mu(x,\lambda)\,.$$

Suppose now that your uncertainty stems from the fact that your measurement apparatus (clock and rods) is fundamentally quantum mechanical: based on $\mathcal{H}_\mathcal{R}$, with Poincaré covariant POVM $\mathsf{E}_\mathcal{R} : \mathrm{Bor}(F(\mathbb{M})) \to \mathcal{E}(\mathcal{H}_\mathcal{R})$, prepared in state $\omega \in \mathscr{D}(\mathcal{H}_\mathcal{R})$. We have a Born probability distribution over possible clocks and rods (tetrads) associated to an oriented quantum reference frame.



Then the **relational local observable** relative to your **quantum reference frame** (QRF) is given by

$$\mathbb{Y}^\mathcal{R}_\omega(\phi) = \phi^{\left(\mu^{\mathsf{E}_\mathcal{R}}_\omega\right)} = \int_{\mathcal{P}^\uparrow_+} U_\mathcal{S}(x,\lambda) \phi U_\mathcal{S}(x,\lambda)^\dagger d\mu^{\mathsf{E}_\mathcal{R}}_\omega(x,\lambda)\,.$$

The relational local observable that one recovers after preparing one's detector (QRF) in a state ω is $\mathbb{Y}^R_\omega(\phi)$, i.e. to every system operator $\phi \in \mathcal{B}(\mathcal{H}_\mathcal{S})$ and detector modelled by the QRF $\mathcal{R} = (U_\mathcal{R}, \mathsf{E}_\mathcal{R}, \mathcal{H}_\mathcal{R})$ which can be prepared in states ω we consider the **relational quantum field**

$$\hat\Phi^\mathcal{R} : \mathscr{D}(\mathcal{H}) \ni \omega \mapsto \mathbb{Y}^R_\omega(\phi) = \int_{\mathcal{P}^\uparrow_+} U_\mathcal{S}(x,\lambda) \phi U_\mathcal{S}(x,\lambda)^\dagger \, d\mu^{\mathsf{E}_\mathcal{R}}_\omega(x,\Lambda) \in \mathcal{B}(\mathcal{H}_\mathcal{S})\,.$$

How does that compare to the usual notion of "quantum field" in physics? The "fuzziness" in the space of inertial reference frames projects to the localisation in spacetime. Consider the disintegration $d\mu^{\mathsf{E}_\mathcal{R}}_\omega(x,\lambda) = d\nu^{\mathsf{E}_\mathcal{R}}_\omega(\lambda \mid x) d\mu^{\mathsf{F}_\mathcal{R}}_\omega(x)$, where $\mathsf{F}_\mathcal{R} : \mathrm{Bor}(\mathbb{M}) \to \mathcal{E}(\mathcal{H}_\mathcal{R})$.

$$\hat\phi^\mathcal{R}(\omega) = \int_\mathbb{M} \hat\phi^\mathcal{R}_\omega(x) d\mu^{\mathsf{F}_\mathcal{R}}_\omega(x) = \int_\mathbb{M} \hat\phi^\mathcal{R}_\omega(x) f^{\mathsf{F}_\mathcal{R}}_\omega(x) d^d x$$

where $f^{\mathsf{F}_\mathcal{R}}_\omega \equiv \frac{d\mu^{\mathsf{F}_\mathcal{R}}_\omega}{d^d x}$ is the **frame smearing function**, and

$$\hat\phi^\mathcal{R}_\omega : \mathbb{M} \ni x \mapsto \int_{\mathcal{L}^\uparrow_+} U_\mathcal{S}(x,\lambda) \phi U_\mathcal{S}(x,\lambda)^\dagger \, d\nu^{\mathsf{E}_\mathcal{R}}_\omega(\lambda \mid x) \in \mathcal{B}(\mathcal{H}_\mathcal{S})$$

is a (pointwise) **relational local quantum field**. It is an integral kernal for $\hat\Phi^\mathcal{R}$ which always exists.

Covariance

$$U_\mathcal{S}(a,\Lambda) \hat\Phi^\mathcal{R}(\omega) U_\mathcal{S}(a,\Lambda)^\dagger = \hat\Phi^\mathcal{R}(U_\mathcal{R}(a,\Lambda)\omega U_\mathcal{R}(a,\Lambda)^\dagger) \quad \left(\sim (a,\Lambda)\cdot \hat\Phi_W(f) = \hat\Phi_W(\{a,\Lambda\}\cdot f)\right)$$

$$U_\mathcal{S}(a,\Lambda) \hat\phi^\mathcal{R}_\omega(x) U_\mathcal{S}(a,\Lambda)^\dagger = \hat\phi^\mathcal{R}_{U_\mathcal{R}(a,\Lambda)\omega U_\mathcal{R}(a,\Lambda)^\dagger}(\Lambda x + a) \quad \left(\sim (a,\Lambda)\cdot \hat\phi(x) = \text{“}\hat\phi(\Lambda x + a)\right)$$

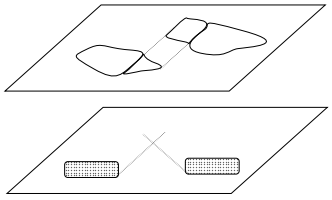
Relational causality

A weak epistemic condition (which is analogous to local commutativity in Wightman QFT) for relational causality is **relational Einstein causality**:

$$\mathrm{supp}\,\mu^{\mathsf{F}_{\mathcal{R}_1}}_{\omega_1} \perp \mathrm{supp}\,\mu^{\mathsf{F}_{\mathcal{R}_2}}_{\omega_2} \Longrightarrow \left[\hat\Phi^{\mathcal{R}_1}(\omega_1), \hat\Phi^{\mathcal{R}_2}(\omega_2)\right] = \left[\hat\Phi^{\mathcal{R}_1}(\omega_1)^\dagger, \hat\Phi^{\mathcal{R}_2}(\omega_2)\right] = 0\,.$$

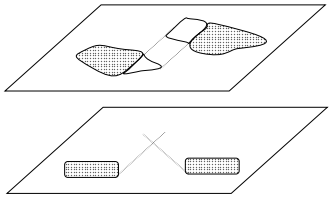
A (stronger) ontic condition (now at the level of kernels) is **weak relational microcausality**:

$$\mathrm{supp}\,\mu^{\mathsf{F}_{\mathcal{R}_1}}_{\omega_1} \perp \mathrm{supp}\,\mu^{\mathsf{F}_{\mathcal{R}_2}}_{\omega_2} \Longrightarrow \left[\hat\phi^{\mathcal{R}_1}_{\omega_1}(x_1), \hat\phi^{\mathcal{R}_2}_{\omega_2}(x_2)\right] = \left[\hat\phi^{\mathcal{R}_1}_{\omega_1}(x_1)^\dagger, \hat\phi^{\mathcal{R}_2}_{\omega_2}(x_2)\right] = 0 \quad \forall x_1, x_2 \in \mathbb{M}\,.$$



An even stronger ontic condition is **strong relational microcausality**:

$$x_1 \perp x_2 \Longrightarrow \left[\hat\phi^{\mathcal{R}_1}_{\omega_1}(x_1), \hat\phi^{\mathcal{R}_2}_{\omega_2}(x_2)\right] = \left[\hat\phi^{\mathcal{R}_1}_{\omega_1}(x_1)^\dagger, \hat\phi^{\mathcal{R}_2}_{\omega_2}(x_2)\right] = 0 \quad \forall \omega_1, \omega_2\,.$$



Finite-precision ($(x-y)^2 > -\sigma$) causality conditions with respect to operationally meaningful preparations of the frame ($\mathfrak{S}_\mathcal{R} \subset \mathscr{D}(\mathcal{H}_\mathcal{R})$), leading to (strong/weak) $(\mathfrak{S}_\mathcal{R}, \sigma)$-(micro)causality can also be written down and a causal covariant relational quantum field can be constructed explicitly [1].

Relational QFT and Algebraic QFT

Let $\mathcal{R}$ be a relativistic QRF, $\mathfrak{S}_\mathcal{R} \subseteq \mathscr{D}(\mathcal{H}_\mathcal{R})$ be convex, $\sigma \geq 0$ and $\mathcal{O}_\mathcal{S} \subseteq \mathcal{B}(\mathcal{H}_\mathcal{S})$ be $(\mathfrak{S}_\mathcal{R}, \sigma)$-causal. Given a subset $U \subseteq \mathbb{M}$, we call

$$\mathcal{A}^{(\mathfrak{S}_\mathcal{R},\sigma)}_{\mathcal{O}_\mathcal{S}}(U) := \{\hat\Phi^\mathcal{R}(\omega) \mid \phi \in \mathcal{O}_\mathcal{S},\, \omega \in \mathfrak{S}_\mathcal{R} \text{ s.t. } \mathrm{supp}\,\mu^{\mathsf{F}_\mathcal{R}}_\omega \subset U\}''$$

a *relational local algebra*. Then

1. (Isotony) For all $U \subseteq V \subseteq \mathbb{M}$, $\mathcal{A}^{(\mathfrak{S}_\mathcal{R},\sigma)}_{\mathcal{O}_\mathcal{S}}(U) \subseteq \mathcal{A}^{(\mathfrak{S}_\mathcal{R},\sigma)}_{\mathcal{O}_\mathcal{S}}(V)$.
2. (Covariance) For all $(a,\Lambda) \in \mathcal{P}^\uparrow_+$ and $U \subseteq \mathbb{M}$, $(a,\Lambda)\cdot \mathcal{A}^{(\mathfrak{S}_\mathcal{R},\sigma)}_{\mathcal{O}_\mathcal{S}}(U) = \mathcal{A}^{((a,\Lambda)\cdot\mathfrak{S}_\mathcal{R},\sigma)}_{\mathcal{O}_\mathcal{S}}((a,\Lambda)\cdot U)$.
3. (Causality) For all $U \perp_\sigma V \subset \mathbb{M}$, $\left[\mathcal{A}^{(\mathfrak{S}_\mathcal{R},\sigma)}_{\mathcal{O}_\mathcal{S}}(U), \mathcal{A}^{(\mathfrak{S}_\mathcal{R},\sigma)}_{\mathcal{O}_\mathcal{S}}(V)\right] = 0$.

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[1] Samuel Fedida and Jan Głowacki. Foundations of Relational Quantum Field Theory I: Scalars, July 2025. arXiv:2507.21601 [quant-ph].

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Making Do and Closing In: A Classification of Effective Field Theories *Qua* Theories

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Radboud Universiteit

I provide a categorization of EFTs *qua* theories, based on effectiveness and renormalizability. It unites previous work on renormalization, connects to metaphysical work on effectiveness, and builds a foundation to answer new questions raised by an EFT paradigm.

Key Concepts for Effective Theories

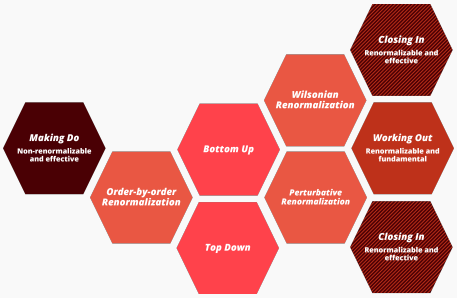
Decoupling Theorem: For composite systems described by a renormalizable theory, there exists a description of the low-energy physics in which the effects of the high-energy physics are included effectively only.

Effectiveness: Deliberate reduction of deg. of freedom to obtain a description valid below some energy Λ, warranted by the decoupling theorem.

Fundamentality: A theory is fundamental if it is applicable at all energies.

Renormalizability: A theory is said to be renormalizable if either (or both) of the Wilsonian or the perturbative renormalization schemes can be applied to obtain a predictive Lagrangian.

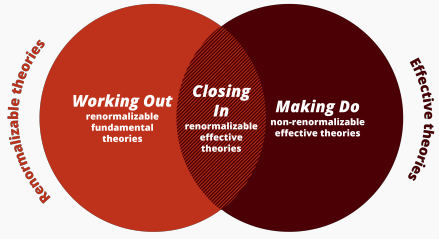
A Visualization of EFT Derivation



How to Derive an EFT in Three Steps

1. **Propose a Lagrangian** based bottom-up on e.g. symmetry principles, conservation laws, experimental data, heuristic assumptions, and, if known, top-down on higher-energy theories.
2. **Regularize the Lagrangian** to remove divergences by introducing a momentum cutoff at an energy Λ or shifting the integral to one over $\delta = d - \epsilon$ dimensions. The integral will then depend on the regulator.
3. **Renormalize the Lagrangian** to remove dependence on the cutoff Λ:
   - *Perturbative Renormalization:* Introduce a finite number of counterterms to rewrite the Lagrangian in such a way that it itself becomes finite.
   - *Wilsonian Renormalization:* Iteratively lower the value of the cutoff from its original value Λ to $\mu < \Lambda$ to find the renormalization group equations for the couplings. The theory is renormalizable if the RG equations hit a fixed point for finite energies.
   - *Order-by-order Renormalization:* If the whole Lagrangian cannot renormalized, it can be truncated at some characteristic energy E_C and renormalization techniques applied to the truncation order by order. For this, combine the introduction of counterterms with a RG-flow analysis.

Three Types of Theories



- Effectiveness and renormalizability are independent
- Theories are defined by whether they are effective and/or renormalizable

⇒ We obtain three types of theories and two types of EFTs

Relation to Other Classifications of Renormalization Schemes and EFTs

	Making Do	Closing In
Bain 2013	continuum EFTs	Wilsonian EFTs
Franklin 2018	real space RG	field theoretic RG
Koberinski and Fraser 2023	full RG	semi RG
Rivat 2019	effective approach	continuum approach

- All make the same point: a contrast of *Closing In* and *Making Do* theories
- They all emphasize the relevance of non-renormalizable effective theories

⇒ The outcome of renormalization is more important than the procedure itself: We should conceive of **EFTs *qua* theories**

Philosophical Implications of Taking EFTs Seriously *Qua* Theories

Zooming in: There are two types of EFT depending on their renormalizability. Renormalizability appears more important than the procedure itself. The two types should be distinguished within the debate.

Zooming out: If EFTs are here to stay, the questions they raise will change from concerning their derivative status to addressing the explanatory or metaphysical status of the theories themselves. To answer such questions, the technical work on renormalization needs to be connected to more conceptual research that explores effectiveness, fundamentality, and reductionism. A perspective on EFTs *qua* theories takes the technical subtleties of EFT derivation and makes them useable for fruitful metaphysical considerations of effectiveness in the future.

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On the Many Ways to Localise a State

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Localisable particles are a main tension between the phenomenology in particle physics and the theoretical description in QFTs due to the various no-go theorems; see Halvorson and Clifton (2002) for an overview.

The **main question**: How to characterise a state as describing a **localised system**, like a particle, under these mathematical restrictions? Several **localisation schemes** are proposed as answers; see Wallace (2001) for *effective localisation* and comparisons to other schemes.

Outline

1. I **propose a new localisation scheme**: *almost localisation*, drawn from Haag-Ruelle scattering.
2. This **reveals a common structure**, which I formalise as the *probe-based localisation schema*.
3. This suggests that whether a field describes a system present in a region is **relative to a choice of parameters of the schema**.

1. New scheme: Almost Localisation

Haag introduces almost local operators in the context of Haag-Ruelle scattering theory (e. g. Haag (1996)). I use them to construct a localisation scheme.

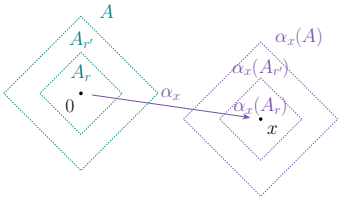
Definition (Haag's almost local operators)

$A \in \mathcal{A}$ is *almost local* iff there is a sequence $A_r \in \mathcal{A}(K_r)$ (double cones of radius r) with

$$\lim_{r\to\infty} r^n \|A - A_r\| = 0 \qquad \forall n \in \mathbb{N}.$$

Almost local operators **approximately behave like strictly local** operators if far separated: denote the spacetime translations by $\alpha(t, \mathbf{x})$; then for C_1, C_2 almost local and any N:

$$\|[\alpha_{(t,\mathbf{x}_1)}(C_1),\, \alpha_{(t,\mathbf{x}_2)}(C_2)]\| < C_N |\mathbf{x}_1 - \mathbf{x}_2|^{-N}$$



Almost local operators, approximated by strictly local operators of growing support regions (dashed diamonds) shifted by spacetime translations.

Let the set $\mathcal{C}_{\text{a.l}}$ consist of all almost local, positive and vacuum-annihilating observables.

Definition (Almost Localisation)

$|\Psi\rangle$ is *almost localised* in $\mathcal{O}$ iff for every $C \in \mathcal{C}_{\text{a.l.}}$

$$\langle\Psi|\, \alpha_{(t,\mathbf{x})}(C)\, |\Psi\rangle$$

falls off superpolynomially in $\operatorname{dist}(\mathbf{x}, \mathcal{O})$.

Note that while $\mathcal{O}$ has to be extended both in space and time to be an open region, it can be made sufficiently thin in time-like direction.

This is itself interesting: it shows that **scattering theory is tightly connected to a notion of localisation**.

2. Probe-Based Localisation Schema

Almost localisation **reveals a conceptual structure** that underlies many of the localisation schemes: probe the state with a class of observables, require the response to "match the vacuum" outside the localisation region.

The blanks to fill in: which probes are relevant to determine what "the field looks like" in $\mathcal{O}$ and what is meant by "matching the vacuum:"

Definition Schema (Probe-based localisation)

An instance of *probe-based localisation* is fixed by

1. a class of probes $\mathcal{O} \mapsto \mathcal{C}(\mathcal{O})$, translation-covariant,
2. a fall-off function $\varepsilon : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ with $\varepsilon(0) \gg 0$.

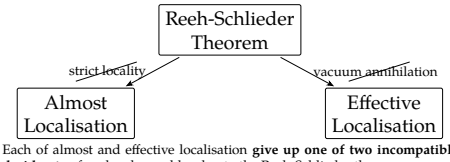
$|\Psi\rangle$ is *localised in* $\mathcal{O}$ iff for all $C \in \mathcal{C}(\mathcal{O})$

$$|\langle\Psi|\, \alpha_{(x)}(C)\, |\Psi\rangle - \langle\Omega|\, \alpha_{(x)}(C)\, |\Omega\rangle| < \varepsilon(\operatorname{dist}(\mathbf{x}, \mathcal{O})).$$

Almost and Wallace's effective localisation are both instances of this schema. Other localisation schemes can be reformulated in this way too; for example Newton-Wigner (with a_{NW} the NW-annihilation operators):

	Probes $\mathcal{C}(\mathcal{O})$	**Fall-off** $\varepsilon(d)$	**Trade-Off**
NW	a_{NW}	0 for $d > 0$	not covariant
Effective	$\mathcal{A}(\mathcal{O})$	$\exp(-\mu d)$	Probes non-zero in $\|\Omega\rangle$
Almost	$\mathcal{C}_{\text{a.l.}}$	$d^{-n}\ \forall n$	Probes not strictly local

Due to Reeh-Schlieder and other no-go theorems, **each of the instances of this schema** has to involve some **trade-off**. Almost localisation and effective localisation in particular can be seen as giving up one of two incompatible desiderata of the probes:



Each of almost and effective localisation **give up one of two incompatible desiderata** of probe observables due to the Reeh-Schlieder theorem.

3. No Schema-Free Localisation

Each instance of the probe-based schema represents a different **trade-off** (Lorentz covariance, sharp regions,…) in light of no-go theorems.

No instance of the schema is privileged by the QFT framework itself. Even if one discards the Newton-Wigner approach due to non-covariance: **effective and almost localisation are two principled responses to** the **Reeh-Schlieder** theorem. One gives up the vacuum annihilation, the other strict locality. Both imply that the region in which a system is localised has unsharp bounds.

It seems that not only the question of what characterises a localised state of a QFT is relative, but the **basic question "Is there a system in $\mathcal{O}$" is not well-posed** without reference to a localisation scheme.

A QFT-based Quantum Universe Without Many Worlds

Arnold Neumaier (University of Vienna)

Presentation at the workshop **Conceptual foundations of QFT**, Vienna, June 1–3, 2026

The Laplacian Quantum Universe

- All physical details in the universe (the only world considered) are uniquely determined by its **universal *-algebra** generated by smeared quantum fields, and its **universal Heisenberg state**.
- The universal Heisenberg state assigns to each X in the universal *-algebra a unique **quantum value** $\langle X \rangle$ (without any a priori statistical connotation).
- The **equations of motion of the universe** are the (nonlinear) **Schwinger–Dyson equations** for the connected N-point functions of the universal Heisenberg state.
- The quantum values of typical **macroscopic quantities** X are directly observable to a good accuracy.
- In particular, this applies to the values of pointer positions in measurements.

⇒ **Measurements have unique outcomes**.
⇒ **Probabilities** and **decoherence** are absent on the level of the universal Heisenberg state, but arise in approximate dynamical models, obtained by the elimination of environmental degrees of freedom.

Approximations

- Models of concrete **physical systems** need approximations by coarse-graining, since the precise state of the universe is unknown.

⇒ **Effective equations of motion**
⇒ **Loss of unitarity**, due to simplifying assumptions underlying the models used, which force a reduced description that misses an important part of what happens in a unitary treatment.

Nonlinearities

- Effective equations of motion are often nonlinear (e.g., Hartree-Fock and Gross–Pitaevskii equations) and sensitive to initial conditions.

⇒ **chaos** and effective **randomness**.

Classical Systems

- For common macroscopic matter, the approximation is done by applying the **maximum entropy principle** to the quantum values of densities, currents, and the stress-energy tensor.

⇒ **fluid dynamics**, **elasticity theory**

Nonlocality

- Nonlocality is due to the fact that the N-point functions entering the equations of motion of the universe are nonlocal for $N > 1$.

⇒ **No hidden variables** are needed.

Acknowledgements

Many thanks to Thomas Stör for extended, interesting discussions, and to Phillip Bachler for help with the poster presentation.

Classical and Quantum Measurements

- A **classical measurement** at some spacetime point x is the reading of the value of some extensive or intensive field, averaged over a mesoscopic neighborhood of x.
- A **quantum measurement** is the reading of a single detector response to a microscopic system, measured according to the
- **Detector Response Principle (DRP):** *Each detection element responds to an incident stationary source with a nonnegative mean rate depending linearly on the density operator of the source. The mean rates sum to the intensity of the field.*

⇒ **POVM**, **Born rule**

State Determination

- A handful of classical measurements at x reveals the state of a classical system close to x.
- A handful of quantum measurements do not reveal anything about the state of the microscopic system. To determine this state one needs **quantum state tomography**, using a large number of quantum measurements.

Interpretation Issues

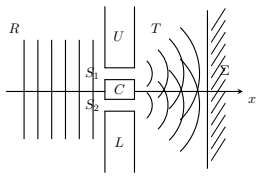
- Every concept is **rigorously defined**, consistent with algebraic QFT.

⇒ Like in geometry, **no further interpretation** is needed.

Further Details

see
`https://arnold-neumaier.at/ms/Laplace.pdf`

The Double Slit Experiment



A Unitary Double Slit Model

In a Laplacian quantum universe, what happens at the screen is adequately handled by the DRP.
But what happens at the barrier?

- We idealize the barrier to be an infinitely extended, infinitesimally thin, and perfectly reflecting polished surface containing a double slit with infinitesimally narrow slits.

⇒ Unitary dynamics for a model universe solely consisting of the free quantum electromagnetic field in the complement of this surface.
⇒ Single photon states satisfy the free Maxwell equations with reflecting boundary conditions.

Mathematical Analysis

- In a 2-dimensional model, a single photon traveling from left to right along a narrow beam of light whose cross section covers the area of the slits is, in the vicinity of the double slit, modeled for $t < 0$ by a traveling wave $F_{\text{in}}(x, y, t) = F(x - ct)$ with $F(x) = 0$ for $x > 0$. It arrives at the barrier at time $t = 0$ and is **reflected everywhere except at the slits**, where we get for $t > 0$ and $x > 0$ a superposition $F_{\text{out}}(x, y, t)$ of two spherical waves.
- If the cross section has width A and the two slits have total width $D \ll A$, only the fraction $p := D/A \ll 1$ of the total energy E of the incident beam passes through the double slit; thus the total energy of the superposition after passing the barrier is only pE.

⇒ In the unitary description, the total energy E is conserved, but **only the fraction pE is available for measurement at the screen**.

Collapse Due to a Reduced Description

- In the region behind the barrier, the reflected beam projects to the vacuum state $|0\rangle$. After normalization we end up in a superposition $\sqrt{1-p}|0\rangle + \sqrt{p}|1\rangle$ of $|0\rangle$ and the normalized 1-photon state $|1\rangle = F_{\text{out}}/\sqrt{p}$.

⇒ The normalized out-state **consists almost only of vacuum**, with only a small admixed fraction of a single photon state, corresponding to the reduced intensity pE of the beam.

- In an ensemble, p can be interpreted as the **Born probability** for finding photons at one of the slits.
- If, as done in textbooks, the vacuum part is ignored, we find a **collapse of the wave function** by projection to the corresponding eigenspace of the position operator, which then propagates further according to the free dynamics with the new initial conditions.

A Single-World Interpretation Suitable for Quantum Cosmology

1. The mathematical formalism of quantum mechanics is sufficient as it stands. No metaphysics needs to be added to it.
2. It is unnecessary to introduce external observers or to postulate the existence of a realm where the laws of classical physics hold.
3. It makes sense to talk about a state for the whole universe.
4. This state never collapses, and hence the universe as a whole is rigorously deterministic.
5. The ergodic properties of laboratory measuring instruments are inessential to its foundations.
6. The statistical interpretation itself need not be imposed a priori.
7. The symbols of quantum mechanics represent reality just as much as those of classical mechanics.
8. The mathematical formalism of the quantum theory is capable of yielding its own interpretation.

(taken almost verbatim from DeWitt 1971, who had claimed this for Everett's many worlds interpretation)

Causal quantum instruments

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Abstract

A notorious problem in quantum theory is the description of temporally or spacetime extended quantum measurements, which is a necessary tool for information extraction. In non-relativistic quantum theory one has to ensure the constraint of operational causality while in spacetime and field theories this becomes even more involved with the demand for locality as well. I discuss a general framework, the positive formalism, that achieves the reconciliation of quantum measurement theory with spacetime theories, the decoherent histories as an instance of it and how the decoherence condition provides us with a way to construct temporal Kraus operators that satisfy operational causality.

Positive formalism

1. Operational quantum theory

1. The states are represented by density operators and correspond to no-input processes. In terms of state spaces, this means that a state is a map $\mathbb{R} \to \mathcal{B}(\mathcal{H})$ where $\mathcal{B}(\mathcal{H})$ is the state space.
2. A completely positive map $E : \mathcal{B}(\mathcal{H}) \to \mathcal{B}(\mathcal{H})$ acts on a state as $E(\rho) = \sum_i K_i \rho K_i^\dagger$.
3. The effects (POVMs) are positive linear functionals $A : \mathcal{B}(\mathcal{H}) \to \mathbb{R}$ and act on the state as $\rho \mapsto \mathrm{tr}(A\rho)$.
4. The trace represents the particular effect of tracing out or "discarding the system". Here we will call it discard state and denote it $\frac{}{\sigma}$.
5. Born rule $P(i) = \frac{\mathrm{Tr}(K_i \sigma K_i^\dagger)}{\mathrm{Tr}(\sigma)}$

An instrument is a family of CP maps $\{E_a\}$ with $\sum_a E_a$ trace-preserving. The probability of outcome a on a state ρ is $p(a|\rho) = \mathrm{tr}[E_a(\rho)] = \mathrm{tr}(E_a \rho)$, where $E_a = \sum_i K_{a,i}^\dagger K_{a,i}$ is the associated POVM element.
Operational causality (in terms of quantum maps): For instruments $\{A_a\}$ followed by $\{B_b\}$ in time, marginalising the later outcomes recovers the earlier statistics, i.e. $\sum_b \mathrm{tr}[(B_b \circ A_a)(\sigma)] = \mathrm{tr}[A_a(\sigma)]$, $\forall \sigma$:



Hence, future operations cannot affect the statistics of past events.

2. Positive formalism

PF is an axiomatic framework that assigns operational quantum structure to spacetime regions and their boundaries.
Assignments. Each geometric object is assigned an algebraic structure: i) Hypersurface: $\Sigma \mapsto \mathcal{B}_\Sigma$ (boundary data space) ii) Region: $M \mapsto \mathcal{P}_M$ (probe space)
Probability map. The basic invariant of a region, $A_M : \mathcal{P}_M \times \mathcal{B}_{\partial M} \longrightarrow \mathbb{R}$, $A_M(P, b) \geq 0$, takes a probe $P \in \mathcal{P}_M$ and boundary data $b \in \mathcal{B}_{\partial M}$ to a non-negative real number.
Probability rule. Conditional probabilities are ratios: $\Pr(P \mid Q) = \frac{A_M(P,b)}{A_M(Q,b)}$.
Composition (gluing). For adjacent regions sharing a hypersurface, $M = M_1 \cup_\Sigma M_2$, $P = P_1 \diamond P_2$, the probability map factorises through Σ: $A_M(P, b) = \sum_i A_{M_1}(P_1, b_1^i) A_{M_2}(P_2, b_2^i)$.
Tensor product. For disconnected hypersurfaces, $\mathcal{B}_{\Sigma_1 \amalg \Sigma_2} \cong \mathcal{B}_{\Sigma_1} \otimes \mathcal{B}_{\Sigma_2}$.

3. Path-integral formulation

The path-integral formulation realises the PF axioms concretely: $\mathcal{B}_{\partial M}$ is built from a Hilbert space $\mathcal{H}_{\partial M}$, probes are realised by insertion of classical observables F and the probability map A_M becomes a bilinear pairing.
State space: Configuration space $K_M = \{\phi : M \to \mathbb{R}\}$ of field configurations; $\psi \in \mathcal{H}_{\partial M}$ is a wave function on ∂M.
Amplitude map:

$$\rho_M(\psi) = \int_{K_M} \mathcal{D}\phi \psi(\phi|_{\partial M}) e^{iS_M(\phi)}$$

By Riesz representation, this bounded linear functional has a unique representative $|\rho\rangle \in \mathcal{H}_{\partial M}$ such that $\rho_M(\psi) = \langle \rho | \psi \rangle$.
Observable map:

$$\rho_M[F](\psi) = \int_{K_M} \mathcal{D}\phi \psi(\phi|_{\partial M}) F(\phi) e^{iS_M(\phi)}$$

The classical observable $F : K_M \to \mathbb{R}$ produces $\rho_M[F]$ with representative vector $|\rho_F\rangle \in \mathcal{H}_{\partial M}$. For $F = 1$ we recover ρ_M.
Observable types. Two classes appear naturally:

- Slice observables: functions of boundary fields on a single hypersurface Σ.
- Spacetime observables: functions of all bulk fields in M.

Probe map. The path-integral realisation of $A_M(P, b)$ is the bilinear pairing

$$\mathcal{P}_M[F](\sigma) := \langle \rho_F | \sigma | \rho_F \rangle = \sum_{k \in I} \rho_M[F](\sigma \zeta_k) \overline{\rho_M[F](\zeta_k)}$$

for $\sigma \in \mathcal{B}_{\partial M}$, with $\{\zeta_k\}_{k \in I}$ an orthonormal basis of $\mathcal{H}_{\partial M}$. Conditional probabilities are then defined as $\Pr(F|F') = \mathcal{P}_M[F](\sigma)/\mathcal{P}_M[F'](\sigma)$.

Properties.

- **Positivity:** $\mathcal{P}_M[F](\sigma) \geq 0$ for $\sigma \geq 0$.
- **Composition law:** $\mathcal{P}_{M_1 \cup M_2}[F_1, F_2](\sigma_{\mathsf{in}} \otimes \sigma_{\mathsf{out}}) = \sum_{j \in J} \mathcal{P}_{M_1}[F_1](\sigma_{\mathsf{in}} \otimes \xi_j) \mathcal{P}_{M_2}[F_2](\xi_j^* \otimes \sigma_{\mathsf{out}})$
- **Locality:** a probe in $R \subset M$ leaves data outside R untouched.
- **Single-time recovery:** on a single hypersurface, $\mathcal{P}_M$ reproduces $\mathrm{tr}(E\rho)$.

4. Spacetime Kraus operator

When M admits a causal direction, the probe map reorganises into a quantum channel.
Causal orientability. Suppose M is time orientable. Then ∂M splits canonically into past and future hypersurfaces $\partial M = \Sigma_{\mathsf{in}} \sqcup \Sigma_{\mathsf{out}}$ and by the tensor product axiom the boundary Hilbert space factors into $\mathcal{H}_{\partial M} = \mathcal{H}_{\Sigma_{\mathsf{in}}} \otimes \mathcal{H}_{\Sigma_{\mathsf{out}}}$. Slice observables on Σ_{in} and Σ_{out} now carry distinct input/output roles.
Choi–Jamiołowski isomorphism. The boundary state $|\rho_F\rangle$ lives in a tensor product, so it corresponds to an operator $|\rho_F\rangle \in \mathcal{H}_{\Sigma_{\mathsf{in}}} \otimes \mathcal{H}_{\Sigma_{\mathsf{out}}} \quad \leftrightarrow \quad K_F : \mathcal{H}_{\Sigma_{\mathsf{in}}} \to \mathcal{H}_{\Sigma_{\mathsf{out}}}$ with matrix elements given by the path integral with fixed boundary field configurations $\xi_{\mathsf{in}}, \xi_{\mathsf{out}}$ on the respective hypersurfaces:

$$\langle \xi_{\mathsf{out}} | K_F | \xi_{\mathsf{in}} \rangle = \int_{\phi|_{\Sigma_{\mathsf{in}}} = \xi_{\mathsf{in}} \phi|_{\Sigma_{\mathsf{out}}} = \xi_{\mathsf{out}}} \mathcal{D}\phi \, F(\phi) e^{iS_M(\phi)}.$$

This is the spacetime Kraus operator induced by M, F, and S_M.
Channel form. K_F acts on input states via $\sigma_{\mathsf{in}} \longmapsto K_F \sigma_{\mathsf{in}} K_F^\dagger$: a sub-normalised completely positive map $\mathcal{B}(\mathcal{H}_{\Sigma_{\mathsf{in}}}) \to \mathcal{B}(\mathcal{H}_{\Sigma_{\mathsf{out}}})$. K_F represents one outcome; the trace $\mathrm{tr}(K_F \sigma_{\mathsf{in}} K_F^\dagger)$ is the probability of that outcome.
Equivalence. K_F and $|\rho_F\rangle$ encode the same data, related by Choi–Jamiołowski reshaping: the Kraus form is the input/output representation; the boundary state is the symmetric representation.

5. Generalisation of operational causality in spacetime

Causality axiom. The OQT axiom "future operations cannot affect past statistics" generalises in PF to: *operations in a region N cannot affect measurements in a region R unless $R \subset J^+(N)$.* Spacetime locality of a probe is encoded in the support of the observable $F(\phi)$.



Example: no signalling outside the light cone. Let $\mathsf{D}_i := \phi(x_i)$. Apply a kick $F = e^{-i\lambda \mathsf{D}_1}$ in region N and measure D_2 in region R. For a coherent boundary state Ξ_ϕ, an explicit calculation gives

$$\mathcal{P}^{\mathsf{pert}}[\mathsf{D}_2|\mathsf{D}_2] \circ \mathcal{P}[F|F](\Xi_\phi) = \mathbf{e}_{\mathsf{L}}(\Xi_\phi) \left[(i\phi(x_2) - \lambda\Delta)(-i\phi(x_2) - \lambda\overline{\Delta}) \right],$$

where $\Delta := w_{\uparrow\downarrow}(x_2, x_1) - w_{\uparrow\uparrow}(x_1, x_2)$ is the imaginary part of the Wightman two-point function. By microcausality, $\Delta = 0$ for all $x_2 \notin J^+(x_1)$. Thus the kick at x_1 leaves no trace at spacelike-separated x_2: PF satisfies the generalised causality axiom, and no faster-than-light signalling is possible.

Operational decoherent histories as an instance of PF

1. Decoherent histories

DH realises PF in a temporal setting: histories are temporal probes, class operators are temporal Kraus operators and the decoherence functional is the probe map with two different observables F, F'.
Sample space of histories. Fine-grained paths $q(t)$ or coarse-grained subsets $h_i \subset \Omega$ of the sample space.
Class operator: the restricted path integral selecting class a,

$$C_a(t'', q''|q', t') = \int_{q'}^{q''} \mathcal{D}q w_a[q] e^{iS[q]},$$

with $w_a[q]$ the characteristic function of paths in class a. It is the analogue of the amplitude map.
Decoherence functional: Temporal analogue of the probe map $\mathcal{P}_M[F_i|F_j]$ for $i \neq j$. For histories h_i, h_j:

$$D(h_i, h_j) = \int_{q(t) \in h_i} \mathcal{D}q' \int_{q(t) \in h_j} \mathcal{D}q' \exp\{i \left(S[q'] - S[q''] \right)\} \delta(q'_j - q''_j) \sigma(q'_0, q''_0).$$

Properties:

(i) Bilinearity (superposition principle) $D(h_i \sqcup h_j, h_k) = D(h_i, h_k) + D(h_j, h_k)$ when h_i and h_j disjoint.
(ii) hermiticity: $D(h_i, h_j) = D^\dagger(h_j, h_i)$
(iii) strong positivity: $D(h_i, h_j)$ is positive for any collection of subsets $\{h_1, h_2, h_3 ... h_n\}$ of Ω
(iv) normalisation: $\sum_{h_i, h_j} D(h_i, h_j) = D(I, I) = 1$

Decoherence condition and probabilities. For a partition $(\cup_j h_j = \Omega,\ h_i \cap h_j = \emptyset)$ with vanishing off-diagonals $D(h_i, h_j) = 0\ \forall i \neq j$, $p(h_i) := D(h_i, h_i)$.

2. Operational histories

DH becomes operationally equivalent to OQT under two conditions:
1. **(Medium) Decoherence:** $D(h_i, h_j) = 0$ for $i \neq j$.
2. **Existence of records:** $\exists \{R_\alpha\}$ orthogonal projectors with $C_\alpha \rho_i C_\beta^\dagger = \delta_{\alpha\beta} R_\alpha C_\alpha \sigma_i C_\alpha^\dagger R_\alpha$.

When both hold, $\{C_\alpha\}$ realise the OQT/PF structure, i.e. $K_\alpha = C_\alpha$, $\Phi(\sigma) = \sum_\alpha C_\alpha \sigma C_\alpha^\dagger$, $E_\alpha = C_\alpha^\dagger C_\alpha$, $\sum_\alpha E_\alpha = \mathbb{I}$, with temporal composition $C_{(\alpha_1, ..., \alpha_n)} = C_{\alpha_n}^{(n)} \cdots C_{\alpha_1}^{(1)}$.
Operational causality in histories. Marginalisation over later events recovers the earlier statistics $p(a_1, \ldots, a_k) = \sum_{a_{k+1}, ..., a_n} p(a_1, \ldots, a_n)$, the temporal version of the OQT causality axiom.
Example. Two-time question $\{q \in A \text{ at } t_1\} \cap \{q \in B \text{ at } t_2\}$:

- **Composite**: $C_{(A,B)} = R_{AB}(t_f) P_B^{(t_2)} U(t_2, t_1) P_A^{(t_1)}$ does not decohere thus it violates causality.
- **Sequential**: $C_{(A,B)} = R_B(t_2) P_B^{(t_2)} R_A(t_1) P_A^{(t_1)}$ decoheres thus it is causal.

3. Temporal Kraus operators from decoherence functional diagonalisation

The decoherence condition determines a family of temporally extended (continuous-family) Kraus operators that satisfy operational causality.
Setup. For an observable $\widehat{F} = \int q dE_F(q)$, consider the continuous-outcome Kraus family $K_\lambda = g(\lambda - \widehat{F})$, $\|g\|_{L^2} = 1$ representing a measurement of $\widehat{F}$ with resolution function g. The decoherence functional on a state ρ is

$$D(\lambda, \lambda') := \int_{\mathbb{R}} dq \int_{\mathbb{R}} dq' \, g(\lambda - q) \overline{g(\lambda' - q')} \, \mathrm{tr}\big(dE_F(q) \sigma dE_F(q')\big).$$

The next step is to impose the decoherence condition $D(\lambda, \lambda') \approx 0$ for $\lambda \neq \lambda'$. However, the decoherence functional depends on both g and ρ and to proceed further approximations are necessary.
Factorisation via slow variation. On the support of g, assume the spectral density $\mu_\sigma(q) := \mathrm{tr}(d\langle E_F(q)\rangle_\sigma)/dq$ varies slowly: $\mu_\sigma(\lambda - u) \approx \mu_\sigma(\lambda)$. Then

$$\boxed{D(\lambda, \lambda') \approx \underbrace{\mu_\sigma(\lambda)}_{\text{state-dep}} \cdot \underbrace{R_g(\lambda - \lambda')}_{\text{instrument}}}$$

where $R_g(\lambda - \lambda') = \int g(\lambda - u) \overline{g(\lambda' - u)} du$ is the autocorrelation of g.
Consequence. Decoherence reduces to a condition on g alone, $R_g(\lambda - \lambda') \approx 0$ for $\lambda \neq \lambda'$ for all states σ in the slow-variation regime.
The next step is to impose constraints to restrict the form of g depending on e.g. bandwidth in the outcome space, resolution, higher moments, smoothness...

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Acknowledgements

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Is the Standard Model an Effective Theory?

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Based on joint work in progress with Michael Miller. Final version not settled; presented content not necessarily agreed with Mike.

Introduction

Since the 1970s, following the work of Wilson and Kogut (1974), particle physics theories have been interpreted as *effective*.

Two questions arise:

1 What does it mean for a theory to be effective?

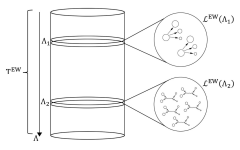
2 Why should we think that the standard model is effective?

What is a 'theory' in particle physics?

Content theories:
A set of quantum fields and symmetries; $\mathcal{T} = (\Phi, \mathcal{S})$.

Effective-field theories:
A cut-off dependent Lagrangian; $\mathcal{L}(\Lambda)$.

Λ-flow theory:
The entire RG group flow of a theory; $T = \{\mathcal{L}(\Lambda) | \Lambda \in \mathbb{R}^+\}$.



The standard model is formally-effective and effectively-interpreted; is it breakdown effective?

Yes! The standard model breaks down for high energies, at least empirically. *But not because it is effectively-interpreted.*

The inadequacy of the standard model is resolvable within the context of effective field theory.

Effectiveness is not the problem

The standard model *is* breakdown-effective

A taxonomy of reasons:
1. UV-incompleteness
2. Gravity
3. Decoupling theorems
4. Theoretical problems/naturalness
5. Empirical anomalies

But none of these imply effective-interpretation must be abandoned.

Reason for Breakdown	Warrant for Breakdown	Effectively Resolvable?
UV-incomplete	Seemingly strong	Yes
Gravity	Strong	Yes
Decoupling	Strong	Yes
Theoretical Problems & Naturalness	Weak / None	Yes
Empirical anomalies	Strong	Yes

Three senses of effectiveness

Formally-effective
A Lagrangian, built from a content theory, that has been regularized and renormalized, $\mathcal{L}(\Lambda)$.

Effectively-interpreted
A formally-effective theory, $\mathcal{L}(\Lambda)$, that has a finite domain of applicability represented by Λ.

Breakdown-effective
A *Λ-flow theory* T such that, at some scale Λ^*, the effective-field theory $\mathcal{L}(\Lambda^*)$ becomes empirically inadequate.

Why effectively-interpret $\mathcal{L}(\Lambda)$?

$\mathcal{L}(\Lambda)$ is incomplete for $\mathrm{k} > \Lambda$. Formally-effective theories are built 'with blurry glasses' by integrating high energies.

Effective-interpretation does not imply breakdown.

Asymptotically safe gravity is an example of a theory that is effectively-interpreted by not necessarily breakdown effective.

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Further information
Please email d.ryder@lse.ac.uk with further questions or for a draft of the paper.

Cluster Decomposition and Entanglement in Perturbative AQFT

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Causal Factorization and the CDP

The Causal Factorization Axiom of perturbative algebraic QFT offers a natural formalization of the Cluster Decomposition Principle, while avoiding the pitfalls of the standard formalization from conventional QFT regarding entangled scattering.

pAQFT as a Philosophical Tool

Philosophy of quantum field theory traditionally proceeds within either 1) the non-perturbative setting of algebraic QFT (AQFT), or 2) the conventional setting of perturbative QFT (pQFT).

- AQFT is a mathematically rigorous setting, but so far only for free theories in 4dim.
- pQFT applies to experimentally-relevant interacting theories, but is beset with mathematical inconsistencies.

There is a gap between the type of empirical objects we study and the applicability of our best mathematical constructions. pAQFT offers a setting to bridge this gap where philosophers can make progress investigating foundational assumptions.

The Cluster Decomposition Principle

It is one of the fundamental principles of physics (indeed, of all science) that experiments that are sufficiently separated in space have unrelated results [...] If this principle were not valid, then we could never make any predictions about any experiment without knowing everything about the universe. - Steven Weinberg [7, 177]

Cluster Decomposition Principle:

Experiments sufficiently separated in space should have unrelated results. In conventional QFT, this principle is formalized as a constraint on the S-matrix ([7])

S-Matrix Factorization (SMF)

if for all $i \neq j$, all of the particles in states α_i and β_i are at a great spatial distance from all of the particles in states α_j and β_j, then:

$$\langle \beta_1 + \beta_2 + \cdots + \beta_N | S | \alpha_1 + \alpha_2 + \cdots + \alpha_N \rangle = \langle \beta_1 | S | \alpha_1 \rangle \langle \beta_2 | S | \alpha_2 \rangle \cdots \langle \beta_N | S | \alpha_N \rangle \quad (1)$$

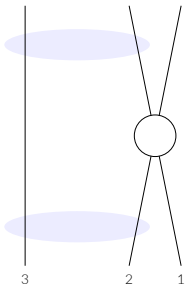
Two Senses of Isolability

Philosophers Porter Williams, John Dougherty, and Michael Miller show that SMF fails for entangled states where the $\alpha_i \to \beta_i$ processes are nonetheless space-like separated from the $\alpha_j \to \beta_j$ processes [9]. For them, SMF conflates two distinct senses of subsystem isolability

1. **Dynamical Influences:** change the statistical distribution of measurements in distinct regions U and V, such that you cannot specify the state in U without including information from V.
2. **Entanglement Influences:** prevent you from specifying a *pure* state for a system in U without information from V, but cannot change the statistical distribution of measurements performed in U given entanglement influences from V.

Doughtery et al. (2024) argue that **the CDP is about dynamical isolability, and thus S-matrix factorization is too strong a condition, as it rules out entangled scattering processes.** They propose to redefine the CDP in terms of the Weinberg Criterion [7][182]

Entangled Scattering



If particles 23 are entangled, $12 \to 12$ will not be independent from $3 \to 3$

$$|\alpha_1\rangle \otimes \frac{1}{\sqrt{2}}(|\alpha_2\rangle \otimes |\alpha_3\rangle + |\alpha_3\rangle \otimes |\alpha_2\rangle) \quad (2)$$

The Weinberg Criterion

A theory will satisfy the cluster decomposition principle only if the interaction Hamiltonian can be expressed as a sum of products of creation and annihilation operators, with all creation operators to the left of annihilation operators, and with coefficients h_{NM} that contain only a single momentum-conservation delta function.

$$H_{\text{int}} = \sum_{N,M} \int dq'_1 \cdots dq'_N \, dp_1 \cdots dp_M \; a^\dagger(q'_1) \cdots a^\dagger(q_M) \times a(q_1) \cdots a(q'_N) \, h_{NM}(\dots) \quad (3)$$

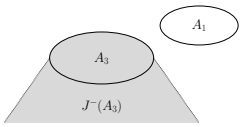
Causality Axioms

In pAQFT, we use the Wightman Propagator of the free-theory to define nets of interacting observables as a formal power series in $\hbar$.

The abstract S-matrix is defined as the generating functional of the time-ordered product $T(A^{\otimes n})$:

$$\mathbf{S}(A) \equiv 1 + \sum_n^\infty \frac{i^n}{n! \hbar^n} \mathcal{T}_n(A^{\otimes n}) \quad (4)$$

$$\equiv T(e_\otimes^{iA/\hbar}) \quad (5)$$



Causal Factorization of T_n

For all $A_1, ..., A_n \in \mathcal{A}$:

$$T_n(A_1, ...A_k, A_{k+1}, ...A_n) = T_k(A_1, ...A_k) \star_{\Delta_H} T_{n-k+1}(A_{k+1}, ...A_n) \quad (6)$$

whenever $A_1, ..., A_k \succ A_{k+1}, ..., A_n$, where $A_1 \succ A_2$ denotes $\text{supp}(A_1) \cap (J^-(\text{supp}(A_2)) \cup \text{supp}(A_2)) = \varnothing$

$\iff$

Causal Factorization of S

whenever $A_1 \succ A_3$

$$\mathbf{S}(A_1 + A_2 + A_3) = \mathbf{S}(A_1 + A_2) \star_{\Delta_H} \mathbf{S}(A_2)^{\star_{-1}} \star_{\Delta_H} \mathbf{S}(A_2 + A_3) \quad (7)$$

Nets of Interacting Observables

For interaction λV and free observable F, the interacting observable $R(e_\otimes^{V/\hbar}, F)$ is given by Bogoliubov's formula:

$$R(e_\otimes^{V/\hbar}, F) = \frac{\hbar}{i} \frac{d}{d\lambda} S_V(F)|_{\lambda=0} \quad (8)$$

where $R_{n,1}(V^{\otimes n}, F)$ is the retarded product corresponding to P, and

$$S_V(F) \equiv S(V)^{\star_{-1}} \star S(V + \lambda F) \quad (9)$$

Fock Space Representations

Using the faithful representation π_0 of on-shell observables by linear operators on Fock space $\mathfrak{F}_0$ for the free real scalar field φ^{op} found by [3] , we know π_0 is an algebra isomorphism $\omega_0(A) = \langle \Omega | \pi_0(A) | \Omega \rangle$, s.t.$\forall A \in \mathcal{A}$:

$$\pi_0(A_1 \star_{\Delta_H} A_2) = \pi_0(A_1)\pi_0(A_2) \quad (10)$$

$$\pi_0(A_1 \cdot A_2) =: \pi_0(A_1)\pi_0(A_2) : \quad (11)$$

I use this algebra isomorphism to show that **the Causal Factorization axiom defines an abstract S-matrix on the abstract algebra of observables, which factorizes iff the Fock space S-matrix operator does. This is a naturally state-independent formalization of cluster decomposition.**

Transition probabilities from initial states $\omega_H(A) = \omega_0(H^* \star A \star H)$ to final states ω_G under interaction $V(g)$, are given by:

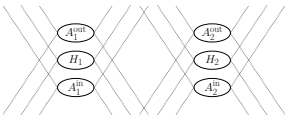
$$\lim_{g(x)\to 1} |\omega_0(G^* \star S(V(g)) \star H)|^2 \quad (12)$$

Entangled Scattering and CF

Consider the Fock space S-matrix for $H = H_1 + ... + H_n$, $H_i \prec\succ H_j$ for all $i \neq j$, evaluated at the states $|\text{in}\rangle = A^{\text{in}} |\Omega\rangle$ and $|\text{out}\rangle = A^{\text{out}} |\Omega\rangle$.

(CFS) Assume that $\text{supp}(A^{\text{in}}) = \mathcal{O}_1^{\text{in}} \cup ... \cup \mathcal{O}_n^{\text{in}}$ and $\text{supp}(A^{\text{out}}) = \mathcal{O}_1^{\text{out}} \cup ... \cup \mathcal{O}_n^{\text{out}}$, and $H_i \prec\succ A_j^{\text{in/out}}$ for mutually spacelike separated regions $\mathcal{O}_i^{\text{in}} \prec\succ \mathcal{O}_j^{\text{in}}$ and $\mathcal{O}_i^{\text{out}} \prec\succ \mathcal{O}_j^{\text{out}}$ for all $i \neq j$.
(SMF) Then, S-Matrix Factorization of transition probabilities holds
(SEP) iff both A^{in} or A^{out} are separable (i.e. not entangled) elements of the tensor product algebras $A(\mathcal{O}_1^{\text{in}}) \otimes ... \otimes A(\mathcal{O}_n^{\text{in}})$ and $A(\mathcal{O}_1^{\text{out}}) \otimes ... \otimes A(\mathcal{O}_n^{\text{out}})$ respectively.

Theorem: (CFS) $\implies$ ((SMF) $\iff$ (SEP))



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Does entanglement through gravity imply gravitons?

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The claim under study

In a thought experiment involving entanglement through gravity, a paradox can arise if quantum fluctuations are ignored. **Causality** and **complementarity** are in conflict [1], which has led to:

The received view

1. **Newtonian entanglement** and consistency with **causality** imply the existence of **gravitons**.
2. There is a regime where Newtonian entanglement can be seen as entanglement through gravitons.

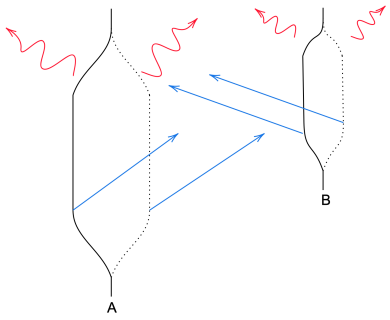


Figure 1. Scalar model of mediated entanglement. The colored arrows correspond to the colored terms in $\hat{\rho}_{AB}$.

The paradox revisited

The paradox involves a one way signalling setup, Fig. 2. The two sides of the paradox correspond to two ways of ignoring quantum fluctuations:

- Set $\Gamma_A = \Gamma_B = 0$: **Complementarity** is violated, B acquires which-path information while A remains coherent.
- Stationary phase approximation: **Causality** is violated, A loses coherence due to B, even though B is in spacelike separation.

For both approximations the which-path information propagates at the speed of light.

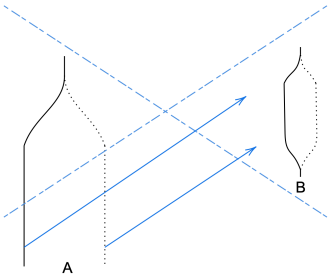
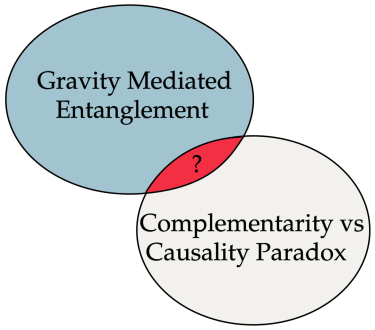


Figure 2. Setup of the paradox.



Scalar model of mediated entanglement

Model linearized gravitational interaction with a scalar field mediator:

- A and B are the two masses,
- each follows a **superposition of trajectories** $\sim |L\rangle + |R\rangle$,
- each mass couples only to the scalar field linearly.

After splitting and recombination, the final state of the two masses

$$\hat{\rho}_{AB} = \frac{1}{4}\begin{pmatrix} 1 & e^{-\Gamma_B} e^{i\varphi_L^A} & e^{-\Gamma_A} e^{i\varphi_L^B} & e^{-(\Gamma_A+\Gamma_B+\Gamma_{AB})} e^{\frac{i}{2}\sum_s(\varphi_s^B+\varphi_s^A)} \\ & 1 & e^{-(\Gamma_A+\Gamma_B-\Gamma_{AB})} e^{\frac{i}{2}\sum_s(\varphi_s^B-\varphi_s^A)} & e^{-\Gamma_A} e^{i\varphi_R^B} \\ & & 1 & e^{-\Gamma_B} e^{i\varphi_R^A} \\ & & & 1 \end{pmatrix}$$

Entangling phases:

- from **retarded potential** of each mass at the position of the other,
- reproduce GME in 'Newtonian' limit [2].

Decoherence terms:

- from **quantum fluctuations** of the mediating field,
- local radiation production $\Gamma_{A/B}$ or correlations harvested from field vacuum Γ_{AB}.

Related through **uncertainty principle** [3, 4]:

$$\Gamma_A \Gamma_B \geq \left((\varphi_R^A - \varphi_R^B) - (\varphi_L^A - \varphi_L^B)\right)^2$$

Our conclusion

1. Entanglement through retarded potentials and consistency with causality imply the existence of gravitons.
2. Only retarded potentials lead to entanglement in this setup, gravitons lead to decoherence.

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Optimization of entanglement harvesting with arbitrary temporal profiles: the limit of second order perturbation theory

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arXiv:2604.06303

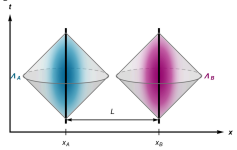
Introduction

The vacuum of a quantum field theory (QFT) contains nonlocal correlations, in particular, entanglement between spacelike separated regions. In entanglement harvesting, we use localized probes that couple locally to the field in order to extract these correlations. Despite several experimental proposals, the resulting entanglement is typically inappreciable and barely large enough to be observed.

In this work, we optimize the protocol in different scenarios by using a Hermite expansion to efficiently compute field propagators smeared with arbitrary switching functions. This allows us to enhance entanglement harvesting by several orders of magnitude, which would take current experimental proposals beyond the regime of second order perturbation theory.

Entanglement Harvesting

The protocol of entanglement harvesting (EH) typically involves two localized probes that interact with a quantum field $\hat{\phi}(\mathsf{x})$ for finite times, in an attempt to extract the entanglement between the regions that they couple to [1, 2]. When the probes become entangled and the communication between them is negligible, one can claim that the entanglement was extracted from the field [3].



We consider EH when two 2-level systems, typically qubits, interact with the field via the local Hamiltonian densities of the form:

$$\hat{h}_{IJ}(\mathsf{x}) = \lambda\left(\mathrm{e}^{\mathrm{i}\Omega t}\Lambda_I(\mathsf{x})\hat{\sigma}_I^+ + \mathrm{e}^{-\mathrm{i}\Omega t}\Lambda_I(\mathsf{x})\hat{\sigma}_I^-\right)\hat{\phi}(\mathsf{x}),$$

where the $\hat{\sigma}_I^\pm$ are the two-level raising and lowering operators acting on the ground and excited state of the detectors, Λ_A, Λ_B are the spacetime regions where the interaction takes place, and $\Omega \geq 0$ is their energy gap.

To leading order in λ, the final state of the detectors $\hat{\rho}_{\mathcal{AB}}$ is determined by the *smeared* local Wightman function and the non-local Feynman propagator:

$$W^{++} = \lambda^2 \int \mathrm{d}V\mathrm{d}V' \mathrm{e}^{-\mathrm{i}\Omega t} \Lambda_{\mathcal{A}}(\mathsf{x}) \mathrm{e}^{\mathrm{i}\Omega t'} \Lambda_{\mathcal{A}}(\mathsf{x}') \langle \hat{\phi}(\mathsf{x}) \hat{\phi}(\mathsf{x}') \rangle,$$

$$G^{++}_{\mathcal{AB}} = \lambda^2 \int \mathrm{d}V\mathrm{d}V' \mathrm{e}^{\mathrm{i}\Omega t} \Lambda_{\mathcal{B}}(\mathsf{x}) \mathrm{e}^{\mathrm{i}\Omega t'} \Lambda_{\mathcal{A}}(\mathsf{x}') \langle \mathcal{T}\hat{\phi}(\mathsf{x}) \hat{\phi}(\mathsf{x}') \rangle,$$

where $\langle\hat{\mathcal{O}}\rangle \equiv \langle 0|\hat{\mathcal{O}}(0)|0\rangle$ denotes the vacuum expectation value and we consider identical comoving detectors.

These propagators allow us to compute the **negativity** $\mathcal{N}(\hat{\rho}_{\mathcal{A}})$, a faithful quantifier of the entanglement between two qubits, to leading order in λ:

$$\mathcal{N}(\hat{\rho}_{\mathcal{A}}) = \max\left(0, |G^{++}_{\mathcal{AB}}| - W^{--}\right) + \mathcal{O}(\lambda^4).$$

Moreover, we can split the Feynman propagator into its symmetric and antisymmetric parts:

$$G^{++}_{\mathcal{AB}} = \frac{1}{2}\left(H^{++}_{\mathcal{AB}} + \mathrm{i}\Delta^{++}_{\mathcal{AB}}\right),$$

where the Hadamard function $H^{++}_{\mathcal{AB}}$ encodes the state dependence and the symmetric propagator $\Delta^{++}_{\mathcal{AB}}$ encodes the exchange of information between the detectors, being zero if they are causally disconnected.

The symmetric propagator allows us to quantify the communication by definining the **Signalling-to-Entanglement Ratio** (SER):

$$\Theta(\hat{\rho}_0) = \begin{cases} \dfrac{\mathcal{N}(\hat{\rho}_0)|_{\Omega\to 0}}{\mathcal{N}(\hat{\rho}_0)} = \dfrac{\frac{1}{2}|\Delta^{++}_{\mathcal{AB}}|}{|G^{++}_{\mathcal{AB}}| - W^{--}}, & \text{if } \mathcal{N}(\hat{\rho}_0) \neq 0, \\ 0, & \text{otherwise.} \end{cases}$$

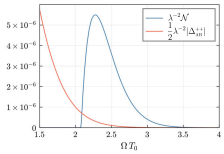
Canonical example

Our baseline example consists of interaction regions defined by Gaussian functions in space and time:

$$\Lambda_{\mathcal{A}}(\mathsf{x}) = \frac{1}{(2\pi\sigma^2)^{\frac{1}{2}}} \mathrm{e}^{-\frac{t^2}{2\sigma^2}} \mathrm{e}^{-\frac{|\mathbf{x}|^2}{2\sigma^2}}, \quad \Lambda_{\mathcal{B}}(\mathsf{x}) = \frac{1}{(2\pi\sigma^2)^{\frac{1}{2}}} \mathrm{e}^{-\frac{t^2}{2\sigma^2}} \mathrm{e}^{-\frac{|\mathbf{x}-\mathbf{L}|^2}{2\sigma^2}}$$

separated by a distance $|\mathbf{L}| = 5T_0$, where T_0 is the Gaussian parameter that controls the duration of the interaction. These choices approximate spacelike separation.

Moreover, assuming that the the detectors' light-crossing time is much smaller than the interaction time T_0 we have $\sigma \ll T_0$. In this regime, one can effectively consider the pointlike limit $\sigma \to 0$. For these parameter choices, closed-form expressions for $G^{++}_{\mathcal{AB}}$ and W^{--} were found in [1].



The figure displays the negativity and signalling estimator of the two detectors as a function of ΩT_0 when $|\mathbf{L}| = 5T_0$. In this example, we can see values of ΩT_0 such that the condition for genuine harvesting $\Theta(\hat{\rho}_0) \ll 1$ is approximately satisfied. For instance, at the peak ($\Omega T_0 \approx 2.3$), we find a negativity of the order of $10^{-6}\lambda^2$, where the signalling amounts to around 5%.

If a setup of this sort is implemented experimentally, one would be able to infer vacuum entanglement between these regions. However, it would be natural to question whether the small order of magnitude would allow any phenomena of this type to be observed in practice.

Arbitrary temporal profiles

To compute $G^{++}_{\mathcal{AB}}$ and W^{--} for arbitrary switching functions, we start by assuming that the smearing functions can be factored as $\Lambda_{\mathcal{A}}(\mathsf{x}) \equiv \chi(t)F(\mathbf{x}), \Lambda_{\mathcal{B}}(\mathsf{x}) \equiv \chi(t)F(\mathbf{x}-\mathbf{L})$, where χ and F are rapidly decreasing L^2 functions. We then consider a truncated basis of Hermite functions $\{h_n(t, T_N)\}_{n\leq N}$ of L^2, rescaled by T_N, so that we can write

$$\chi(t) = \sum_{n=0}^{N} c_n h_n(t, T_N) \equiv \sum_{n=0}^{N} c_n h_{n,N}(t)$$

where $c_n = \sqrt{T_0}\langle h_n, \chi\rangle_{L^2}$ and T_0 is a parameter with units of time, ensuring that $\chi(t)$ is dimensionless. By doing so, we can now write the relevant propagators as **simple matrix products**:

$$G^{++}_{\mathcal{AB}} = \mathbf{c}^{\mathsf{T}}\mathsf{G}\mathbf{c}, \quad W^{--} = \mathbf{c}^{\mathsf{T}}\mathsf{W}\mathbf{c},$$

where $\mathbf{c}$ is the vector with components c_n and G and W are matrices with components:

$$G_{nm} \sim \left[H_n\left(\frac{\mathrm{d}}{\mathrm{d}\alpha}\right) H_m\left(\frac{\mathrm{d}}{\mathrm{d}\beta}\right) G(\alpha,\beta)\right]\Big|_{\alpha=\beta=0},$$

where H_n and H_m are Hermite polynomials, and $G(\alpha,\beta)$ is a generating function. By making the choice:

$$F(\mathbf{x}) = \frac{\mathrm{e}^{-\frac{|\mathbf{x}|^2}{2\sigma^2}}}{(2\pi\sigma^2)^{3/2}}, \quad h_{n,N}(t) = \frac{\pi^{-1/4}}{\sqrt{2^n n! T_N}} H_n(t/T_N)\,\mathrm{e}^{-\frac{t^2}{2T_N^2}}$$

for the spatial smearing and the switching functions, we are able to find closed-form expressions for $G(\alpha,\beta)$.

The computation of the negativity for a given switching function (characterized in this setup by a vector $\mathbf{c}$) becomes the matrix product $\widetilde{\mathcal{N}}(\mathbf{c}) = |\mathbf{c}^{\mathsf{T}}\mathsf{G}\mathbf{c}| - \mathbf{c}^{\mathsf{T}}\mathsf{W}\mathbf{c}$ or, if we impose the norm of our switching function to be equal to that of the Gaussian (to ensure a fair comparison with the canonical example),

$$\widetilde{\mathcal{N}}(\mathbf{c}) = \sqrt{\pi}\frac{|\mathbf{c}^{\mathsf{T}}\mathsf{G}\mathbf{c}| - \mathbf{c}^{\mathsf{T}}\mathsf{W}\mathbf{c}}{\mathbf{c}^{\mathsf{T}}\mathbf{c}}$$

We can now compute the negativity and signalling for **any switching function** of our choice, and this will allow us to optimize the protocol by maximizing $\widetilde{\mathcal{N}}(\mathbf{c})$ under different signalling constraints.

Conclusions

We optimized the protocol of entanglement harvesting and, in different setups, our method can pinpoint the optimal switching function that maximizes entanglement extraction under different signalling constraints. By recasting the computation of propagators as simple matrix products, we are able to produce harvesting plots for arbitrary functions in fractions of a second.

Although we restricted our analysis to switching functions in Minkowski spacetime, the method can also be extended to spatial smearing functions and the basis expansion method can, in principle, be generalized to arbitrary spacetimes, where the causal and geometric structure can substantially affect the correlations available for harvesting.

Up to this point, entanglement harvesting has only been treated perturbatively, resulting in an infinitesimal amount of extracted entanglement, and posing an experimental challenge in practice. However, **extrapolating our conclusions to recent experimental proposals pushes the protocol beyond leading order perturbation theory**, and perhaps even beyond the perturbative regime altogether. Our results make it clear that a non-perturbative treatment is the only way forward in the path to make entanglement harvesting useful in quantum information and quantum computing.

References

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Results

Spacelike separated regions

In this case, we find the **optimal temporal profile** for the interaction for harvesting between **spacelike separated regions**. While, in principle, the method presented does not allow one to have genuine spacelike separation due to the tails of Hermite functions at a given truncation N, we can approximate compact support arbitrarily well by tuning the parameter T_N in the expression of $\chi_n(t)$.

Indeed, if we choose $T_N \sim \lfloor L/2 \rfloor / \sqrt{2N+1}$, the Hermite functions span the space of compactly supported functions with $t \in [-L/2, L/2]$ for large N, ensuring causal disconnection. In this case, we can safely set the signalling term to zero in $G^{++}_{\mathcal{AB}}$ and the negativity becomes

$$\widetilde{\mathcal{N}}^+(\mathbf{c}) = \sqrt{\pi}\frac{|\mathbf{c}^{\mathsf{T}}\mathsf{H}\mathbf{c}| - \mathbf{c}^{\mathsf{T}}\mathsf{W}\mathbf{c}}{\mathbf{c}^{\mathsf{T}}\mathbf{c}}$$

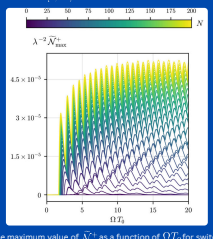
which is defined in [3] as the *harvested negativity*, as it considers that all correlations stem from the state dependent part of the Feynman propagator. We can now simply maximize $\widetilde{\mathcal{N}}^+$, leading to an optimization problem which can be rewritten as

$$\widetilde{\mathcal{N}}^+_{\max} = \max_{\theta} \max_{\mathbf{c}^{\mathsf{T}}\mathbf{c}=1} \mathsf{M}^+(\theta),$$

where

$$\mathsf{M}^+(\theta) = \frac{1}{2}\left(\mathrm{Re}(\mathsf{H})\cos\theta + \mathrm{Im}(\mathsf{H})\sin\theta\right) - \mathrm{Re}(\mathsf{W}).$$

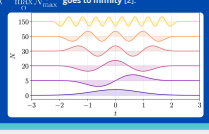
Thus, the problem reduces to finding the **largest eigenvalue** of the real symmetric matrix $\mathsf{M}^+(\theta)$ for each $\theta \in [0, 2\pi)$.



Here, we plot the maximum value of $\widetilde{\mathcal{N}}^+$ as a function of ΩT_0 for switching functions within the subspaces $\{h_n(t, T_N)\}_{n\leq N}$ for detectors separated by a distance of $|\mathbf{L}| = 5T_0$, which allows us to compare these results with the canonical example.

As N increases, so does the number peaks and their height. Even with spacelike separation between the detectors, the optimization yields negativities of the order of $10^{-1}\lambda^2$ for $N = 200$, **one order of magnitude larger** than the canonical example.

Moreover, we plot the optimal switching function for each truncation and we note that, as N increases, the **optimal functions** become more **oscillatory**. These results show that, in the large N limit, our optimal switching functions become super oscillatory and $\lambda^{-2}\max_{\mathbf{c}}\widetilde{\mathcal{N}}^+_{\max}$ **goes to infinity** [2].



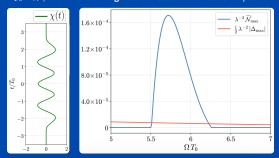
Scan this code and generate, in fractions of a second, entanglement harvesting plots for a switching function of your choice!

https://entanglement-harvesting-explorer.streamlit.app/

Approximately causally disconnected regions

In this case, we slightly increase the interaction times for the detectors, which allows us to recover much larger negativities at the cost of introducing a small level of signalling

Indeed, we now slightly **increase** the **compact support parameter** T_N, until the *SER* reaches values of $\sim 5\%$ and we are now able to recover negativities of the order of $10^{-4}\lambda^2$, **two orders of magnitude** over the canonical example.

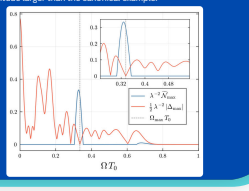


Non-communicating causally connected regions

Even when the detectors are fully causally connected, it is possible that, for a specific choice of parameters, they do not communicate at leading order. In this case one cannot use the detectors to quantify entanglement between spacelike separated regions, as they couple to overlapping field degrees of freedom. Nevertheless, if the signalling-to-entanglement ratio is exactly zero, one can still claim that the detectors became entangled due to pre-existing entanglement in the field.

Indeed, it is possible to find vectors $\mathbf{c}$ such that $\mathbf{c}^{\mathsf{T}}\Delta^{++}_{\mathcal{AB}}\mathbf{c} = 0$. This constraint prevents this optimization from being recast as a simple eigenvalue problem, so we instead use numerical methods to find the optimal switching function.

Here, we plot the negativity and signalling estimator for the optimal switching function with $N = 2\Omega T = 2L = 10T_0$ and $L = 5T_0$ increasing the interaction time by a factor of 10. As expected, this setup has large signalling for most values of Ω. However, at $\Omega_{\max}T_0 \approx 0.536$, the communication cancels exactly, corresponding to a point where $\Delta^{++}_{\mathcal{AB}}$ changes in sign, while the negativity reaches $10^{-1}\lambda^2$, five order of magnitude larger than the canonical example.



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