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Conceptual Foundations of QFT, Vienna, 1-3 June 2026

[AGP-RV 1] Albert Georg Passegger, RV, Rev. Math. Phys. **37** (2025) 2430009

[AGP-RV 2] Albert Georg Passegger, RV, Ann. H. Poincaré (2025)
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About this talk

- QFT on Minkowski spacetime — Klein-Gordon field
- (Thermal) Equilibrium and Non-equilibrium situations for time-evolutions w.r.t. Lorentz boosts, or inertial frames
 - ★ KMS \longleftrightarrow Tomita-Takesaki Δ^{it} ★
- Couple to *Unruh-de Witt* detectors
- Provide arguments for: Asymptotic behaviour of detectors is characteristic of the *folium* of certain states
- \longrightarrow possible characterization of close-to-equilibrium states beyond KMS ?
- Relation to other generalizations of the KMS condition

- 4-dim Minkowski spacetime $M = \mathbb{R}^{1,3}$
- Klein-Gordon operator $K = \square + m^2 : C_0^\infty(M) \rightarrow C_0^\infty(M)$
- Causal Green-bidistribution $G_c : C_0^\infty(M) \times C_0^\infty(M) \rightarrow \mathbb{C}$,
 $G_c(Kf, h) = 0 = G_c(f, Kh)$,
 $G_c(f, h) = 0$ if $\text{supp}(f)$ and $\text{supp}(h)$ causally separated
- Abstract field operators for the CCR $*$ -algebra A_{CCR} , generated by $\mathbf{1}$ and $\Phi(f)$, $f \in C_0^\infty(M)$, with relations:

$f \mapsto \Phi(f)$ is \mathbb{C} -linear

$$\Phi(f)^* = \Phi(\bar{f})$$

$$[\Phi(f), \Phi(h)] = iG_c(f, h)\mathbf{1}$$

$$\Phi(Kf) = 0$$

A state $\omega : \mathbf{A}_{\text{CCR}} \rightarrow \mathbb{C}$ is a linear functional with $\omega(\mathbf{1}) = 1$ such that the

$$w_n^{(\omega)}(f_1, \dots, f_n) = \omega(\Phi(f_1) \cdots \Phi(f_n))$$

fulfill the distribution- and positivity-properties of *Wightman functions*

→

Can pass to the GNS-Wightman representation π_ω with

- Hilbert space \mathcal{H}_ω
- GNS vector Ω_ω
- Wightman domain $\mathcal{D}_\omega = \pi_\omega(\mathbf{A}_{\text{CCR}})\Omega$

$$\varphi_\omega(f) = \pi_\omega(\Phi(f))$$

$$\langle \Omega_\omega, \varphi_\omega(f_1) \cdots \varphi_\omega(f_n) \Omega_\omega \rangle = \omega(\Phi(f_1) \cdots \Phi(f_n))$$

The $\varphi_\omega(f)$, for real f , are symmetric operators on \mathcal{D}_ω .

If \mathcal{D}_ω is a domain of analytic vectors for the $\varphi_\omega(f)$ (f real), then

$$\mathbf{w}_{\omega,f}(t) = e^{it\varphi_\omega(f)} \quad (t \in \mathbb{R})$$

is a continuous unitary group with selfadjoint generator $\varphi_\omega(f)$.

This always holds if ω is a quasifree state (determined by two-point function $w_2^{(\omega)}$)

The structure of the C^* -algebra $\mathfrak{A}(N)$ generated by $\mathbf{w}_{\omega,t}(f)$, $\text{supp}(f) \subset N$ ($N \subset M$) is independent of ω .

→

- $N_1 \subset N_2 \implies \mathfrak{A}(N_1) \subset \mathfrak{A}(N_2)$
- $[\mathfrak{A}(N_1), \mathfrak{A}(N_2)] = \{0\}$ if N_1 and N_2 causally separated
- $L \in \text{Iso}_+^\uparrow(M) \implies$

$\alpha_L(\varphi_\omega(f)) = \varphi_\omega(f \circ L^{-1})$ induces automorphism

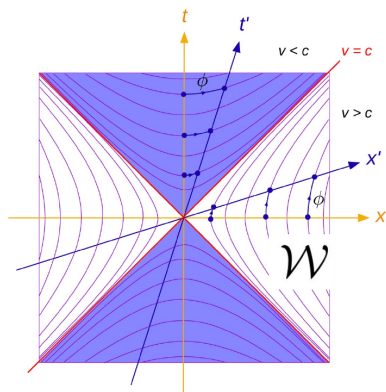
$$\alpha_L : \mathfrak{A}(M) \rightarrow \mathfrak{A}(M) \text{ with } \alpha_L(\mathfrak{A}(N)) = \mathfrak{A}(L(N)), \quad \alpha_{L_1} \circ \alpha_{L_2} = \alpha_{L_1 L_2}$$

Let $\{L^t\}_{t \in \mathbb{R}}$ be a 1-parametric, smooth subgroup of $\text{Iso}_+^{\uparrow}(M)$, with **future-pointing timelike orbits** on a maximal invariant region \mathcal{N} , either:

- ◇ **inertial time-shifts** $L^t(x) = x + ut$ for u timelike unit vector, $\mathcal{N} = \mathcal{M}$, or
- ◇ **boosts** $L^t = \Lambda_t =$ Lorentz-boosts in x^1 -direction, $\mathcal{N} = \mathcal{W}$

$$\Lambda_t = \begin{pmatrix} \cosh(t) & \sinh(t) & 0 & 0 \\ \sinh(t) & \cosh(t) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathcal{W} = \{(x^0, x^1, x^2, x^3) : 0 < |x^0| < x^1\}$$



Write $\alpha^t = \alpha_{Lt}$.

Let ω be a state on $\mathfrak{A}(\mathcal{N})$

- ω is a **ground state** w.r.t. $\{\alpha^t\}_{t \in \mathbb{R}}$ if

$$\frac{1}{i} \frac{d}{dt} \omega(A^* \alpha^t(A)) \Big|_{t=0} \geq 0 \quad (A \in \mathcal{A}(\mathcal{N}))$$

- ω is a **KMS state at inverse temperature** $\beta > 0$ w.r.t. $\{\alpha^t\}_{t \in \mathbb{R}}$ if

$$\widehat{F}_{A,B}(-k) = e^{\beta k} \widehat{F}_{B,A}(k) \quad \text{with} \quad F_{A,B}(t) = \omega(A \alpha^t(B)) \quad (A, B \in \mathcal{A}(\mathcal{N}))$$

- ω is an (inertial) **vacuum state** if $\omega \circ \alpha_L = \omega$ and if ω is a ground state for all inertial time-shifts.

Bisognano-Wichmann theorem:

The restriction of the inertial vacuum state ω_{vac} to $\mathfrak{A}(\mathcal{W})$ is a KMS state at $\beta = 2\pi$ for $\{\alpha^t\}_{t \in \mathbb{R}}$ in the case

$$L^t = \Lambda_t = \text{Lorentz boosts leaving } \mathcal{W} \text{ invariant}$$

In the GNS representation $(\mathcal{H}_{\text{vac}}, \pi_{\text{vac}}, \Omega_{\text{vac}})$ of ω_{vac} , this means that

$$\pi_{\text{vac}}(\alpha_{\Lambda_t}(\mathbf{A})) = \Delta^{i2\pi t} \pi_{\text{vac}}(\mathbf{A}) \Delta^{-i2\pi t} \quad (\mathbf{A} \in \mathfrak{A}(\mathcal{W}))$$

where $\{\Delta^{it}\}_{t \in \mathbb{R}}$ is the Tomita-Takesaki modular group of

$$\mathbf{A}_{\text{vac}}(\mathcal{W}) = \pi_{\text{vac}}(\mathfrak{A}(\mathcal{W}))'' \quad \text{and} \quad \Omega_{\text{vac}}$$

The **detector** is a quantum mechanical 2-level system. Observable algebra: $\mathbf{A}_D = M_{2 \times 2}(\mathbb{C})$. In its “ground state representation”:

$$\mathcal{H}_{D,gr} = \mathbb{C}^2, \quad H_D = \begin{pmatrix} E & 0 \\ 0 & 0 \end{pmatrix},$$

$$\text{KMS states : } \omega_{D,\beta}(\cdot) = \frac{1}{\text{Tr}(e^{-\beta H_{D,gr}})} \text{Tr}(e^{-\beta H_{D,gr}}(\cdot))$$

$$\alpha_{D,t}(\mathbf{a}) = U_{D,t} \mathbf{a} U_{D,t}^*, \quad U_{D,t} = e^{iH_{D,gr}t}$$

Every state on \mathbf{A}_D is given by a density matrix in the ground state representation, and the folia of states coincide.

For any state σ on \mathbf{A}_D invariant under $\alpha_{D,t}$, denote by $\mathbf{L}_{D,\sigma}$ the Liouvillian,

$$e^{i\mathbf{L}_{D,\sigma}t} \pi_\sigma(\mathbf{a}) e^{-i\mathbf{L}_{D,\sigma}t} = \pi_\sigma(\alpha_{D,t}(\mathbf{a})), \quad e^{i\mathbf{L}_{D,\sigma}t} \Omega_\sigma = \Omega_\sigma$$

Coupling Dynamics of Detector and Field

Suppose that detector and field are both prepared as KMS states with inverse temperature β w.r.t. their intrinsic (uncoupled) dynamics.

The **coupled dynamics** is defined on $\mathcal{H}_{D,\beta} \otimes \mathcal{H}_{F,\beta}$ with **interacting Liouvillian**

$$\mathbf{L}_\lambda^{int} = \mathbf{L}_{D,\beta} \otimes \mathbf{1} + \mathbf{1} \otimes \mathbf{L}_{F,\beta} + \lambda \mathbf{V}_{int} \quad \text{with the interaction term}$$

$$\mathbf{V}_{int} = Q - (J_D \otimes J_F)Q(J_D \otimes J_F), \quad Q = \mathbf{g} \otimes \varphi_\beta(h)$$

- $\lambda > 0$ is a parameter regulating coupling strength
- $J_{D/F}$ are the modular conjugations of $\mathbf{A}_{D/F,\beta}, \Omega_{D/F,\beta}$
- $\mathbf{g} \in M_{2 \times 2}(\mathbb{C})$ is hermitean, with **non-diagonal entries** $\neq 0$
- h is a test-function with suitable “spectral properties” and “small support” (to approximate a “pointlike detector”)

Two approaches towards asymptotic behavior of coupled dynamics

1 – Asymptotic Detector Response Rate (perturbative)

The **transition probability/unit of time** of passing from the ground state $\left(\begin{pmatrix} 0 \\ 1 \end{pmatrix}, (\cdot) \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right)$ prepared at asymptotically early times to the excited state $\left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}, (\cdot) \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right)$ at asymptotically late times for a **pointlike detector coupled to the quantum field along the worldline** $\gamma_Y(\tau) = L^\tau y$ at lowest order in perturbation theory (expanding the dynamics of L_λ^{int} in a Dyson series for small λ) is given by

$$\mathcal{T}(0 \rightarrow E) = \lambda^2 \left| \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{g} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right) \right|^2 R(E),$$

$$R(E) = \lim_{T \rightarrow \infty} \lim_{h \rightarrow \delta_y} \int_{-T}^T \int_{-T}^T \frac{e^{i(\tau - \tau')E}}{2T} \omega_{F,\beta} \left(\alpha_F^T(\varphi_\beta(h)) \alpha_F^{\tau'}(\varphi_\beta(h)) \right) d\tau d\tau'$$

provided the quantum field is prepared in the state $\omega_{F,\beta}$.

τ = proper time parameter for $\gamma_Y(\tau)$

For $L^\tau =$ Lorentz boosts, $y = (0, y_1, 0, 0) \in \mathcal{W}$ it is known:

- **Unruh effect** [Unruh (1976)]

$$R(E) = \frac{E}{2\pi(e^{\beta_y E} - 1)}, \quad \beta_y = 2\pi \cdot y_1$$

($1/y_1 =$ proper acceleration of γ_y) if $\omega_{F,\beta} = \omega_{\text{vac}}|_{\mathfrak{A}(\mathcal{W})}$ (i.e. $\beta = 2\pi$)

- This holds more generally for a dense set of states in the **folium** of ω_{vac} , where

$$\tilde{\omega} \in \mathbf{folium}(\omega_{\text{vac}}) \quad \text{if} \quad \tilde{\omega}(A) = \text{Tr}(\pi_{\omega_{\text{vac}}}(A)\varrho)$$

for any **density matrix** ϱ on $\mathcal{H}_{\omega_{\text{vac}}}$

- This indicates: The “thermalization” of the detector is a consequence of the coupling over infinite duration of time.

It doesn't reflect local “thermal” properties of the states in the folium of ω_{vac} .

2 – Asymptotic State of the Detector

The **thermalization property**, or **RTE property**, of the coupled system:

Let the state ω on $\mathcal{A}_D \otimes \mathcal{A}_F$ be a KMS state w.r.t. the interacting dynamics $\alpha_{\lambda,t}^{int}$ induced by L_{λ}^{int} .

The state ω fulfills the **thermalization property** if for every state $\tilde{\omega} \in \mathbf{folium}(\omega)$, it holds that $\lim_{t \rightarrow \infty} \tilde{\omega}(\alpha_{\lambda,t}^{int}(A)) = \omega(A)$ ($A \in \mathcal{A}_D \otimes \mathcal{A}_F$).

Thermalization theorem

The KMS state ω fulfills the thermalization property iff $|\Omega_{\omega}\rangle\langle\Omega_{\omega}|$ is the only proper eigenprojector of L_{λ}^{int} (“Koopmanism”)

[Bach, Dereziński, Fröhlich, Jakšić, Merkli, Pillet, ...]

Note: In this case, the test function h in the interaction term $Q = g \otimes \varphi_{\beta}(h)$ is in $C_0^{\infty}(M)$ and cannot be shrunk to δ_y ! (No “pointlike” detector)

For the coupled detector and field, one can show that there is a β -KMS state ω for $\alpha_{\lambda,t}^{int}$ on $\mathbf{A}_D \otimes \mathbf{A}_F$, with $\omega \in \mathbf{folium}(\omega_{D,\beta} \otimes \omega_{F,\beta})$,
fulfilling the thermalization property

- for sufficiently small λ
- for suitable test-functions h

The partial state of ω on \mathcal{A}_D approximates $\omega_{D,\beta}$ as $\lambda \rightarrow 0$.

Established for the following cases:

♠ $\omega_{F,\beta} = \omega_{\text{vac}}|_{\mathfrak{A}_F(\mathcal{W})}$ ($\beta = 2\pi$),
 α_F^t corresponding to Lorentz boosts [de Bièvre & Merkli 2006]

♠ $\omega_{F,\beta} = \omega^{(u)}$ = inertial KMS state, $\alpha_F^t = \alpha_t^{(u)}$ “detector at rest in a heat bath” [AGP-RV 1], variation of results on “Pauli-Fierz systems” [Jakšić & Pillet]

Consequence: Preparing the coupled system at $t = 0$ in any state $\tilde{\omega} \in \mathbf{folium}(\omega)$, evolved under the interacting dynamics the partial detector state will at asymptotically large times be found approximately in the β -KMS state (for small λ).

The result rests on RTE arguments for KMS states $\omega = \omega_\lambda^{int}$ under perturbations of the dynamics – here in the form of adding the interaction term λV_{int} to the uncoupled dynamics.

These RTE arguments establish the existence of KMS states for the interacting dynamics.

[Araki, Bach, Dereziński, Fröhlich, Jakšić, Merkli, Pillet, Pinamonti et al., . . .]

The property $\omega_\lambda^{int} \in \mathbf{folium}(\omega_{D,\beta} \otimes \omega_{F,\beta})$ is key – the states ω_λ^{int} are constructed as states in the GNS representation of $\omega_{D,\beta} \otimes \omega_{F,\beta}$.

Let $\alpha_t^{(v)}$ ($t \in \mathbb{R}$) denote the time-shift automorphism group w.r.t.

$$L^t x = x + tv \quad (v = \text{future-pointing vector})$$

Let $\omega^{(u)}$ denote β_u -KMS state on $\mathfrak{A}(M)$ w.r.t. $\alpha_t^{(u)}$.

Then $\omega^{(u)}$ is **not** a KMS state w.r.t. $\alpha_t^{(v)}$ if $u \not\parallel v$.

This is due to redshift/blueshift effects when an observer moves with constant velocity relative to an inertial heat bath.

- With $u = (1, 0, 0, 0)$, $v = (1, v, 0, 0)$,

$$R_v(E) = \frac{\sqrt{1-v^2}}{4\pi\beta v} \ln \left(\frac{1 - e^{-\beta E d_+}}{1 - e^{-\beta E d_-}} \right)$$

$$d_{\pm} = \frac{\sqrt{1 \pm v}}{\sqrt{1 \mp v}}, \quad \alpha_F^t = \alpha_t^{(v)}, \quad \omega_{F,\beta} = \omega^{(u)}, \quad \beta = \beta_u$$

[Costa and Matsas (1995), also AGP-RV 2]

- $\text{folium}(\omega^{(u)}) \cap \text{folium}(\omega^{(v)}) = \emptyset$
- $\text{folium}(\omega_{D,\beta} \otimes \omega_{F,\beta}^{(u)}) \cap \text{folium}(\omega_{D,\beta} \otimes \omega_{F,\beta}^{(v)}) = \emptyset$

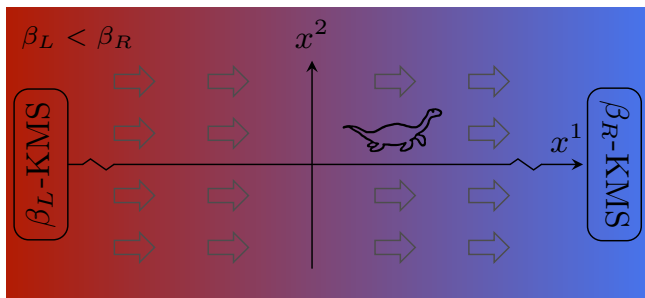
[AGP-RV1] (generalizes and synthesizes earlier related results [Sewell, Ojima, Narnhofer & Thirring, Herman & Takesaki, Buchholz & Ojima, Buchholz & Longo ...])

Therefore, a KMS state ω_λ^{int} for an interaction perturbation of the uncoupled dynamics $\alpha_{D,t} \otimes \alpha_{F,t}^{(v)}$ **cannot be constructed in $\text{folium}(\omega_{D,\beta} \otimes \omega_{F,\beta}^{(u)})$ using RTE-arguments.**

This is an (indirect) argument why the observer that moves with constant velocity w.r.t. an inertial heat bath will not find the comoving detector in an (asymptotic) thermal equilibrium.

Non-equilibrium steady states (NESS) are stationary states of a (quantum) system of degrees of freedom in contact with several thermal reservoirs (with respect to the same inertial time-direction $u = (1, 0, 0, 0)$) held at distant locations.

Simple case for the quantized Klein-Gordon field: “Hotter” reservoir with β_L at $x^1 = -\infty$, “colder” reservoir with β_R at $x^1 = +\infty$, $\beta_L < \beta_R$.



A NESS with the detector “NESSie” coupled to it

The corresponding heat flux j is compensated by a **comoving velocity** $v_N = (1, v_N, 0, 0)$ with

$$v_N = \frac{4(\beta_L^4 + \beta_R^4) - \sqrt{(\beta_R^4 + 7\beta_L^4)(\beta_L^4 + 7\beta_R^4)}}{3(\beta_R^4 - \beta_L^4)}$$

(massless Klein-Gordon field)

[AGP-RV 2]

Asymptotic detector response rates, $\omega_F = \omega_{\text{NESS}}^{(u)}$, $\alpha_F^t = \alpha_t^{(v)}$

* **Detector at rest w.r.t. the reservoirs**, $v = u$:

$$R(E) = \frac{E}{4\pi(e^{\beta_L E} - 1)} + \frac{E}{4\pi(e^{\beta_R E} - 1)}$$

This is the average of the response rate of detectors at rest w.r.t. heat baths at inverse temperatures β_L and β_R . The detector is not sensitive to the heat flux.

- * **Detector moving with velocity** v , coupled to NESS w.r.t. u

$$R(E) = \text{function of } (\beta_L, \beta_R, v, u)$$

Worth noting:

- For $v = v_N$ (detector comoving with heat flux), $R(E)$ does **not** show a thermal response rate. Inertial motion w.r.t. the asymptotic heat reservoirs of the NESS noticed by the detector. Again, a sign that the detector does not couple to the heat flux.
- The response rate (for general v) is **not** just the average of the response rates of detectors in inertial motion w.r.t. heat baths at inverse temperatures β_L and β_R .

- Unruh-de Witt detectors assume asymptotic response rates characteristic of thermal/equilibrium/non-equilibrium properties when coupled to corresponding states, and/or moving w.r.t. reservoirs defining such states
- The asymptotic rates or detector states are not assumed for specific states, but for (dense) sets of states \rightarrow characteristic for a folium of states.
- Offers the opportunity to use asymptotic detector response rates as “generalizations of the KMS condition” for folia of states. Singling out of particular states by time-scales within which asymptotic response rate is attained.
- Caveats: Detector coupling is model-specific, local properties of states not directly tested

- Unruh-de Witt detectors with varying λ and E can be viewed as (pre-stage to) **quantum reference frames**. Refinement of this idea could be used to avoid model-dependence of the UdW detector.
- **Local thermal equilibrium states (LTE)** [Buchholz, Ojima, Roos (2002)]

Locally near every spacetime point x , such states behave like $\omega^{(u)}$, β_u KMS states w.r.t. *local thermal observables* S_x , $u = u(x)$, $\beta_u = \beta_u(x)$

For quantized Klein-Gordon field, this is equivalent to a **local KMS condition** [Gransee, Pinamonti, RV (2015)]

These conditions refer to (approximate) thermal equilibrium conditions at very small scales.