

# Local Measurement and the Type III Property in Algebraic QFT

Noel Swanson

University of Delaware

Conceptual Foundations of QFT  
ÖAW, June 1-3, 2026

# QMT Frameworks for AQFT

## Fewster-Verch (2020)

- ▶ Abstract  $*$ -algebras
- ▶ Curved or flat spacetime
- ▶ Locally covariant AQFT, pAQFT, Haag-Kastler AQFT, ...
- ▶ Local time-slice property (primitive causality) plays key role

# QMT Frameworks for AQFT

## Fewster-Verch (2020)

- ▶ Abstract  $*$ -algebras
- ▶ Curved or flat spacetime
- ▶ Locally covariant AQFT, pAQFT, Haag-Kastler AQFT, ...
- ▶ Local time-slice property (primitive causality) plays key role

## Okamura-Ozawa (2016)

- ▶ Concrete von Neumann algebras
- ▶ Flat spacetime
- ▶ Haag-Kastler AQFT + DHR
- ▶ Type III + split properties play key roles

# QMT Frameworks for AQFT

## Fewster-Verch (2020)

- ▶ Abstract  $*$ -algebras
- ▶ Curved or flat spacetime
- ▶ Locally covariant AQFT, pAQFT, Haag-Kastler AQFT, ...
- ▶ Local time-slice property (primitive causality) plays key role

## Okamura-Ozawa (2016)

- ▶ Concrete von Neumann algebras
- ▶ Flat spacetime
- ▶ Haag-Kastler AQFT + DHR
- ▶ Type III + split properties play key roles

## Converse Projects?

FV: coupling + probe observables/states  $\Rightarrow$  induced system observables

OO: system observables  $\Rightarrow$  coupling + probe observables

## Type III Property

Local VN algebras  $\mathfrak{R}(O)$  are generically type  $\text{III}_1$  factors

VN factor is type. . .

- I if  $\exists$  minimal projection
- II if  $\nexists$  minimal projection,  $\exists$  nonzero finite projection
- III if  $\nexists$  nonzero finite projection

# Type III Property

Local VN algebras  $\mathfrak{A}(O)$  are generically type  $\text{III}_1$  factors

VN factor is type. . .

- I if  $\exists$  minimal projection
- II if  $\nexists$  minimal projection,  $\exists$  nonzero finite projection
- III if  $\nexists$  nonzero finite projection
  - ▶  $\text{III}_0$  if Connes spectrum =  $\{0, 1\}$
  - ▶  $\text{III}_\lambda$ ,  $0 < \lambda < 1$  if Connes spectrum =  $\{\lambda^n\} \cup \{0\}$
  - ▶  $\text{III}_1$  if Connes spectrum =  $\mathbb{R}^+$

# Type III Property

Local VN algebras  $\mathfrak{A}(O)$  are generically type  $\text{III}_1$  factors

VN factor is type. . .

- I if  $\exists$  minimal projection
- II if  $\nexists$  minimal projection,  $\exists$  nonzero finite projection
- III if  $\nexists$  nonzero finite projection
  - ▶  $\text{III}_0$  if Connes spectrum =  $\{0, 1\}$
  - ▶  $\text{III}_\lambda$ ,  $0 < \lambda < 1$  if Connes spectrum =  $\{\lambda^n\} \cup \{0\}$
  - ▶  $\text{III}_1$  if Connes spectrum =  $\mathbb{R}^+$

**Type  $\text{III}_1$  factors are wild!** No pure normal states, no faithful semi-finite normal trace, maximally noncommutative, orbit of any normal state under inner automorphisms norm dense, every state “equally mixed”

## Type III Property

Local VN algebras  $\mathfrak{A}(O)$  are generically type  $\text{III}_1$  factors

VN factor is type. . .

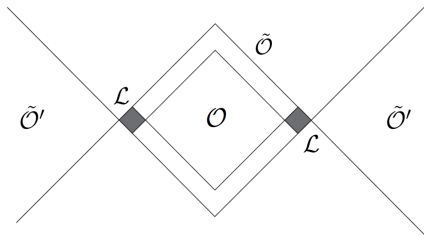
- I if  $\exists$  minimal projection
- II if  $\nexists$  minimal projection,  $\exists$  nonzero finite projection
- III if  $\nexists$  nonzero finite projection
  - ▶  $\text{III}_0$  if Connes spectrum =  $\{0, 1\}$
  - ▶  $\text{III}_\lambda$ ,  $0 < \lambda < 1$  if Connes spectrum =  $\{\lambda^n\} \cup \{0\}$
  - ▶  $\text{III}_1$  if Connes spectrum =  $\mathbb{R}^+$

**Type  $\text{III}_1$  factors are wild!** No pure normal states, no faithful semi-finite normal trace, maximally noncommutative, orbit of any normal state under inner automorphisms norm dense, every state “equally mixed”

$\Rightarrow$  Revise our concept of superposition, transition probabilities, state preparation, subsystem decomposition, VN entropy . . .

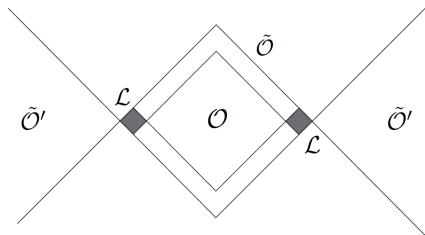
## Split Property

$\exists$  type I factor  $\mathfrak{N}$  s.t.  $\mathfrak{K}(O) \subset \mathfrak{N} \subset \mathfrak{K}(\tilde{O})$



## Split Property

$\exists$  type I factor  $\mathfrak{N}$  s.t.  $\mathfrak{K}(O) \subset \mathfrak{N} \subset \mathfrak{K}(\tilde{O})$



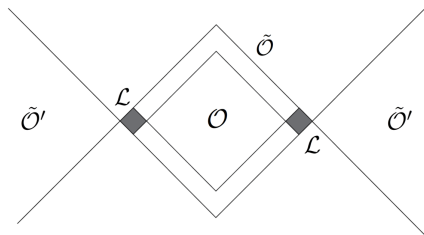
$\Rightarrow \mathfrak{K}(O)$  is **hyperfinite** and **injective**, i.e.,

$$\mathfrak{K}(O) = \overline{\bigcup_{\alpha} \mathfrak{N}_{\alpha}}^{uw} \quad (1)$$

and  $\exists$  a conditional expectation  $\mathcal{E} : B(\mathcal{H}) \rightarrow \mathfrak{K}(O)$  (not normal!)

## Split Property

$\exists$  type I factor  $\mathfrak{N}$  s.t.  $\mathfrak{K}(O) \subset \mathfrak{N} \subset \mathfrak{K}(\tilde{O})$



$\Rightarrow \mathfrak{K}(O)$  is **hyperfinite** and **injective**, i.e.,

$$\mathfrak{K}(O) = \overline{\bigcup_{\alpha} \mathfrak{N}_{\alpha}}^{uw} \quad (1)$$

and  $\exists$  a conditional expectation  $\mathcal{E} : B(\mathcal{H}) \rightarrow \mathfrak{K}(O)$  (not normal!)

Reeh-Schlieder Theorem  $\Rightarrow \mathfrak{K}(O)$  and  $\mathfrak{K}(\tilde{O}')$  statistically independent

# Standard QMT

- von Neumann (1932)

Discrete, nondegenerate observables

(i) Repeatability  $\Rightarrow \rho \rightarrow \rho' = \sum P_i \rho P_i$

(ii)  $\rho \rightarrow \rho'$  compatible with system-apparatus  $H$ -dynamics

# Standard QMT

- von Neumann (1932)  
Discrete, nondegenerate observables
  - (i) Repeatability  $\Rightarrow \rho \rightarrow \rho' = \sum P_i \rho P_i$
  - (ii)  $\rho \rightarrow \rho'$  compatible with system-apparatus  $H$ -dynamics
- Lüders (1951)  
Degenerate observables + (i)
- Nakamura and Umegaki (1962), Arveson (1967)  
No-Go for continuous observables + (i)

# Standard QMT

- von Neumann (1932)  
Discrete, nondegenerate observables
  - (i) Repeatability  $\Rightarrow \rho \rightarrow \rho' = \sum P_i \rho P_i$
  - (ii)  $\rho \rightarrow \rho'$  compatible with system-apparatus  $H$ -dynamics
- Lüders (1951)  
Degenerate observables + (i)
- Nakamura and Umegaki (1962), Arveson (1967)  
No-Go for continuous observables + (i)
- Davies and Lewis (1970)  
Drop repeatability + Instruments (general  $\rho \rightarrow \rho' +$  POVMs)

# Standard QMT

- von Neumann (1932)  
Discrete, nondegenerate observables
  - (i) Repeatability  $\Rightarrow \rho \rightarrow \rho' = \sum P_i \rho P_i$
  - (ii)  $\rho \rightarrow \rho'$  compatible with system-apparatus  $H$ -dynamics
- Lüders (1951)  
Degenerate observables + (i)
- Nakamura and Umegaki (1962), Arveson (1967)  
No-Go for continuous observables + (i)
- Davies and Lewis (1970)  
Drop repeatability + Instruments (general  $\rho \rightarrow \rho' +$  POVMs)
- Kraus (1971)  
CP instruments + (ii)

# Standard QMT

- von Neumann (1932)  
Discrete, nondegenerate observables
  - (i) Repeatability  $\Rightarrow \rho \rightarrow \rho' = \sum P_i \rho P_i$
  - (ii)  $\rho \rightarrow \rho'$  compatible with system-apparatus  $H$ -dynamics
- Lüders (1951)  
Degenerate observables + (i)
- Nakamura and Umegaki (1962), Arveson (1967)  
No-Go for continuous observables + (i)
- Davies and Lewis (1970)  
Drop repeatability + Instruments (general  $\rho \rightarrow \rho' +$  POVMs)
- Kraus (1971)  
CP instruments + (ii)
- Ozawa (1984)  
Continuous observables + Representation Theorem

# CP Instruments

VN algebra  $\mathfrak{M}$ , Hilbert space  $\mathcal{H}$ , Borel space  $(S, \mathcal{F})$

An **instrument**  $\mathcal{I} : \mathcal{F} \rightarrow P(\mathfrak{M}_*)$  (positive linear maps on  $M_*$ ) satisfies

(i)  $\langle \mathcal{I}(S)\rho, 1 \rangle = \langle \rho, 1 \rangle$

(ii)  $\langle \mathcal{I}(\cup_i \Delta_i)\rho, A \rangle = \sum_i \langle \mathcal{I}(\Delta_i)\rho, A \rangle$

$\forall A \in \mathfrak{M}, \rho \in \mathfrak{M}_*$ , mutually disjoint  $\{\Delta_i\}$  of  $\mathcal{F}$

# CP Instruments

VN algebra  $\mathfrak{M}$ , Hilbert space  $\mathcal{H}$ , Borel space  $(S, \mathcal{F})$

An **instrument**  $\mathcal{I} : \mathcal{F} \rightarrow P(\mathfrak{M}_*)$  (positive linear maps on  $M_*$ ) satisfies

(i)  $\langle \mathcal{I}(S)\rho, 1 \rangle = \langle \rho, 1 \rangle$

(ii)  $\langle \mathcal{I}(\cup_i \Delta_i)\rho, A \rangle = \sum_i \langle \mathcal{I}(\Delta_i)\rho, A \rangle$

$\forall A \in \mathfrak{M}, \rho \in \mathfrak{M}_*$ , mutually disjoint  $\{\Delta_i\}$  of  $\mathcal{F}$

Interpretation:

$$\langle \mathcal{I}(\Delta)\rho, 1 \rangle = \|\mathcal{I}(\Delta)\rho\| = \Pr(x \in \Delta | \rho) \quad (2)$$

State change:

$$\rho \rightarrow \rho_{\mathcal{I}}^{\Delta} = \frac{\mathcal{I}(\Delta)\rho}{\|\mathcal{I}(\Delta)\rho\|} \quad (3)$$

## CP Instruments (cont.)

The dual map  $\mathcal{I}^*(\Delta) : \mathcal{F} \rightarrow P_n(\mathfrak{M})$  is defined by

$$\langle \rho, \mathcal{I}^*(\Delta)A \rangle = \langle \mathcal{I}(\Delta)\rho, A \rangle \quad (4)$$

$$\forall \rho \in \mathfrak{M}_*, A \in \mathfrak{M}$$

## CP Instruments (cont.)

The dual map  $\mathcal{I}^*(\Delta) : \mathcal{F} \rightarrow P_n(\mathfrak{M})$  is defined by

$$\langle \rho, \mathcal{I}^*(\Delta)A \rangle = \langle \mathcal{I}(\Delta)\rho, A \rangle \quad (4)$$

$$\forall \rho \in \mathfrak{M}_*, A \in \mathfrak{M}$$

We can give an alternative characterization of instruments as mappings  $\mathcal{I}(A, \Delta) : \mathfrak{M} \times \mathcal{F} \rightarrow \mathfrak{M}$ , defined by

$$\mathcal{I}(A, \Delta) = \mathcal{I}^*(\Delta)A \quad (5)$$

$$\forall A \in \mathfrak{M}$$

## CP Instruments (cont.)

The dual map  $\mathcal{I}^*(\Delta) : \mathcal{F} \rightarrow P_n(\mathfrak{M})$  is defined by

$$\langle \rho, \mathcal{I}^*(\Delta)A \rangle = \langle \mathcal{I}(\Delta)\rho, A \rangle \quad (4)$$

$$\forall \rho \in \mathfrak{M}_*, A \in \mathfrak{M}$$

We can give an alternative characterization of instruments as mappings  $\mathcal{I}(A, \Delta) : \mathfrak{M} \times \mathcal{F} \rightarrow \mathfrak{M}$ , defined by

$$\mathcal{I}(A, \Delta) = \mathcal{I}^*(\Delta)A \quad (5)$$

$$\forall A \in \mathfrak{M}$$

A **CP instrument** is an instrument s.t.  $\mathcal{I}(\Delta)$  (equiv  $\mathcal{I}(A, \Delta)$ ) is completely positive  $\forall \Delta \in \mathcal{F}$

# Measurement Processes

PVM or POVM  $X$ , Hilbert space  $\mathcal{H}$ , Borel space  $(S, \mathcal{F})$

A **measurement process** for  $X$  is a 4-tuple  $\mathbb{M} = (\mathcal{K}, \sigma, \tilde{X}, U)$

- apparatus Hilbert space  $\mathcal{K}$
- normal “ready” state  $\sigma$
- pointer PVM or POVM  $\tilde{X} : \mathcal{F} \rightarrow B(\mathcal{K})$
- unitary system-apparatus dynamics  $U : \mathcal{H} \otimes \mathcal{K} \rightarrow \mathcal{H} \otimes \mathcal{K}$

satisfying  $\forall \Delta \in \mathcal{F}$

$$\text{Tr}[\rho X(\Delta)] = \text{Tr}[U(\rho \otimes \sigma)U^* \tilde{X}(\Delta)] \quad (6)$$

# Measurement Processes

PVM or POVM  $X$ , Hilbert space  $\mathcal{H}$ , Borel space  $(S, \mathcal{F})$

A **measurement process** for  $X$  is a 4-tuple  $\mathbb{M} = (\mathcal{K}, \sigma, \tilde{X}, U)$

- apparatus Hilbert space  $\mathcal{K}$
- normal “ready” state  $\sigma$
- pointer PVM or POVM  $\tilde{X} : \mathcal{F} \rightarrow B(\mathcal{K})$
- unitary system-apparatus dynamics  $U : \mathcal{H} \otimes \mathcal{K} \rightarrow \mathcal{H} \otimes \mathcal{K}$

satisfying  $\forall \Delta \in \mathcal{F}$

$$\text{Tr}[\rho X(\Delta)] = \text{Tr}[U(\rho \otimes \sigma)U^* \tilde{X}(\Delta)] \quad (6)$$

State change:

$$\rho \rightarrow \rho_{\mathbb{M}}^{\Delta} = \frac{E_{\mathcal{K}}[U(\rho \otimes \sigma)U^*(1 \otimes \tilde{X}(\Delta))]}{\text{Tr}[\rho X(\Delta)]} \quad (7)$$

# Ozawa Representation Theorem

A CP instrument  $\mathcal{I}$  is **X-compatible** if  $X(\Delta) = \mathcal{I}(1, \Delta)$

Two  $X$ -measurement processes  $\mathbb{M}_1, \mathbb{M}_2$  are **statistically equivalent** if  $\forall A \in B(\mathcal{H})$

$$\text{Tr}[\rho_{\mathbb{M}_1}^\Delta A] = \text{Tr}[\rho_{\mathbb{M}_2}^\Delta A] \quad (8)$$

# Ozawa Representation Theorem

A CP instrument  $\mathcal{I}$  is  **$X$ -compatible** if  $X(\Delta) = \mathcal{I}(1, \Delta)$

Two  $X$ -measurement processes  $\mathbb{M}_1, \mathbb{M}_2$  are **statistically equivalent** if  $\forall A \in B(\mathcal{H})$

$$\text{Tr}[\rho_{\mathbb{M}_1}^\Delta A] = \text{Tr}[\rho_{\mathbb{M}_2}^\Delta A] \quad (8)$$

## Representation Theorem (Ozawa 1984)

*There is a 1-1 correspondence between statistical equivalence classes of measurement processes for  $X$  and  $X$ -compatible CP instruments on  $\mathfrak{M} = B(\mathcal{H})$ , given by:*

$$\mathcal{I}(A, \Delta) = (id \otimes \sigma)[U^*(A \otimes \tilde{X}(\Delta))U] \quad (9)$$

# Ozawa Representation Theorem

A CP instrument  $\mathcal{I}$  is **X-compatible** if  $X(\Delta) = \mathcal{I}(1, \Delta)$

Two  $X$ -measurement processes  $\mathbb{M}_1, \mathbb{M}_2$  are **statistically equivalent** if  $\forall A \in B(\mathcal{H})$

$$\text{Tr}[\rho_{\mathbb{M}_1}^\Delta A] = \text{Tr}[\rho_{\mathbb{M}_2}^\Delta A] \quad (8)$$

## Representation Theorem (Ozawa 1984)

*There is a 1-1 correspondence between statistical equivalence classes of measurement processes for  $X$  and  $X$ -compatible CP instruments on  $\mathfrak{M} = B(\mathcal{H})$ , given by:*

$$\mathcal{I}(A, \Delta) = (\text{id} \otimes \sigma)[U^*(A \otimes \tilde{X}(\Delta))U] \quad (9)$$

Key fact used in proof: Irreducible normal representations of  $B(\mathcal{H})$  are unique up to unitary equivalence. **Theorem fails if  $\mathfrak{M}$  is not a type I factor!**

# Okamura-Ozawa Framework (2016)

- 1.) Generalized Representation Theorem: If  $\mathfrak{M} \neq B(\mathcal{H})$ , not every CP instrument has a measurement process that implements it. But every CP instrument with a certain **normal extension property** does.
- 2.) Density Theorem: If  $\mathfrak{M}$  is injective, then  $\text{CPIInst}_{NEP}$  is dense in  $\text{CPIInst}$ .
- 3.) Application to AQFT in DHR picture (Local Instrument Theorem)

# Normal Extension Property

Outcomes of measurement processes described by abelian VN algebra  $L^\infty(S)$  of bounded random variables on  $(S, \mathcal{F})$ . Role of CP instruments is to link  $\mathfrak{M}$  to  $L^\infty(S)$ .

# Normal Extension Property

Outcomes of measurement processes described by abelian VN algebra  $L^\infty(S)$  of bounded random variables on  $(S, \mathcal{F})$ . Role of CP instruments is to link  $\mathfrak{M}$  to  $L^\infty(S)$ .

Fact 1: If  $\mathfrak{M}$  is a type I factor, every CP instrument on  $\mathfrak{M}$  extends uniquely to a unital normal CP map  $\tilde{\psi}_I : \mathfrak{M} \overline{\otimes} L^\infty(S) \rightarrow \mathfrak{M}$ .

# Normal Extension Property

Outcomes of measurement processes described by abelian VN algebra  $L^\infty(S)$  of bounded random variables on  $(S, \mathcal{F})$ . Role of CP instruments is to link  $\mathfrak{M}$  to  $L^\infty(S)$ .

Fact 1: If  $\mathfrak{M}$  is a type I factor, every CP instrument on  $\mathfrak{M}$  extends uniquely to a unital normal CP map  $\tilde{\psi}_{\mathcal{I}} : \mathfrak{M} \overline{\otimes} L^\infty(S) \rightarrow \mathfrak{M}$ .

Fact 2: If  $\mathfrak{M}$  is a general VN algebra, every CP instrument on  $\mathfrak{M}$  extends uniquely to a unital CP map  $\psi_{\mathcal{I}} : \mathfrak{M} \otimes_{bin} L^\infty(S) \rightarrow \mathfrak{M}$ .

# Normal Extension Property

Outcomes of measurement processes described by abelian VN algebra  $L^\infty(S)$  of bounded random variables on  $(S, \mathcal{F})$ . Role of CP instruments is to link  $\mathfrak{M}$  to  $L^\infty(S)$ .

Fact 1: If  $\mathfrak{M}$  is a type I factor, every CP instrument on  $\mathfrak{M}$  extends uniquely to a unital normal CP map  $\tilde{\psi}_I : \mathfrak{M} \overline{\otimes} L^\infty(S) \rightarrow \mathfrak{M}$ .

Fact 2: If  $\mathfrak{M}$  is a general VN algebra, every CP instrument on  $\mathfrak{M}$  extends uniquely to a unital CP map  $\psi_I : \mathfrak{M} \otimes_{bin} L^\infty(S) \rightarrow \mathfrak{M}$ .

## Normal Extension Property (NEP)

*A CP instrument has the normal extension property if there exists a unital normal CP map  $\tilde{\psi}_I : \mathfrak{M} \overline{\otimes} L^\infty(S) \rightarrow B(\mathcal{H})$  s.t.  $\tilde{\psi}_I|_{\mathfrak{M} \otimes_{bin} L^\infty(S)} = \psi_I$ .*

# Generalized Representation and Density Theorems

## Generalized Representation Theorem (Okamura & Ozawa 2016)

*There is a 1-1 correspondence between statistical equivalence classes of measurement processes and CP instruments with the NEP on  $\mathfrak{M}$ .*

Note: This requires that  $(id \otimes \sigma)[U^*(A \otimes \tilde{X}(\Delta))U] \in \mathfrak{M}$  in the definition of a measurement process, ensuring that the instrument  $\mathcal{I}$  defined by (9) is CP on  $\mathfrak{M}$ .

# Generalized Representation and Density Theorems

## Generalized Representation Theorem (Okamura & Ozawa 2016)

*There is a 1-1 correspondence between statistical equivalence classes of measurement processes and CP instruments with the NEP on  $\mathfrak{M}$ .*

Note: This requires that  $(id \otimes \sigma)[U^*(A \otimes \tilde{X}(\Delta))U] \in \mathfrak{M}$  in the definition of a measurement process, ensuring that the instrument  $\mathcal{I}$  defined by (9) is CP on  $\mathfrak{M}$ .

## Density Theorem (Okamura & Ozawa 2016)

*If  $\mathfrak{M}$  is injective, then for every CP instrument  $\mathcal{I}$  there exists a net  $\{I_\alpha\}$  of CP instruments with NEP s.t.  $\mathcal{I}_\alpha \xrightarrow{uw} \mathcal{I}$*

# Repeatability Loophole

Fact 3: If  $\mathfrak{M} = B(\mathcal{H})$ , then every (weakly) repeatable instrument is discrete.

# Repeatability Loophole

Fact 3: If  $\mathfrak{M} = B(\mathcal{H})$ , then every (weakly) repeatable instrument is discrete.

Fact 4: If  $\mathfrak{M}$  is a general VN algebra, a (weakly) repeatable instrument has the NEP iff it is discrete.

# Repeatability Loophole

Fact 3: If  $\mathfrak{M} = B(\mathcal{H})$ , then every (weakly) repeatable instrument is discrete.

Fact 4: If  $\mathfrak{M}$  is a general VN algebra, a (weakly) repeatable instrument has the NEP iff it is discrete.

⇒ Type I case: no continuous observable has a (weakly) repeatable instrument.

⇒ Non-Type I case: there can be continuous observables with (weakly) repeatable instruments which lack the NEP.

⇒ If  $\mathfrak{M}$  is injective these can be approximated by nets of instruments with the NEP.

# Application to AQFT

Haag-Kastler Axioms

(Isotony, Microcausality, Covariance, Vacuum, Spectrum)

+ Split Property

+ DHR Axioms

(Haag Duality, Separability, Property B)

# Application to AQFT

Haag-Kastler Axioms

(Isotony, Microcausality, Covariance, Vacuum, Spectrum)

+ Split Property

+ DHR Axioms

(Haag Duality, Separability, Property B)

**DHR Selection Condition:** Physically salient representations are quasiequivalent to vacuum representation in  $O'$ , the causal complement some double cone  $O$ .

# Application to AQFT

Haag-Kastler Axioms

(Isotony, Microcausality, Covariance, Vacuum, Spectrum)

+ Split Property

+ DHR Axioms

(Haag Duality, Separability, Property B)

**DHR Selection Condition:** Physically salient representations are quasiequivalent to vacuum representation in  $O'$ , the causal complement some double cone  $O$ .

Charge sectors = quasiequivalence classes of DHR-rep

$\Rightarrow$  Reconstruct field algebra + global gauge group

(Casimir invariants label charge sectors)

# Local Instruments

Consider  $O$ -localized excitations in the DHR picture

Question: When does an instrument  $\mathcal{I}$  on  $\mathfrak{K}(O)$  extend to an instrument on global algebra  $\mathfrak{K}$  with nice localization properties?

# Local Instruments

Consider  $O$ -localized excitations in the DHR picture

Question: When does an instrument  $\mathcal{I}$  on  $\mathfrak{K}(O)$  extend to an instrument on global algebra  $\mathfrak{K}$  with nice localization properties?

A **local instrument** for  $(\mathfrak{K}, O, \tilde{O})$  is an instrument on  $\mathfrak{K}$  satisfying

$$\mathcal{I}(AB, \Delta) = \mathcal{I}(A, \Delta)B \quad (10)$$

$\forall \Delta \in \mathcal{F}, A \in \mathfrak{K}, B \in \mathfrak{K}(\tilde{O}')$ , and

$$\mathcal{I}(C, \Delta) \in \mathfrak{K}(O) \quad (11)$$

$\forall \Delta \in \mathcal{F}, C \in \mathfrak{K}(O)$

# Existence of Local Instruments

## Local Instrument Theorem (Okamura & Ozawa 2016)

*If the Haag-Kastler axioms + split property + DHR axioms hold, then every CP instrument  $\mathcal{I}$  on  $\mathfrak{K}(O)$  with the NEP extends to a local CP instrument  $\tilde{\mathcal{I}}$  for  $(\mathfrak{K}, O, \tilde{O})$ .*

# Existence of Local Instruments

## Local Instrument Theorem (Okamura & Ozawa 2016)

*If the Haag-Kastler axioms + split property + DHR axioms hold, then every CP instrument  $\mathcal{I}$  on  $\mathfrak{K}(O)$  with the NEP extends to a local CP instrument  $\tilde{\mathcal{I}}$  for  $(\mathfrak{K}, O, \tilde{O})$ .*

## Corollary

*Because  $\mathfrak{K}(O)$  is injective, the density, generalized representation, and local instrument theorems entail that for every CP instrument  $\mathcal{I}$  on  $\mathfrak{K}(O)$  and any  $\epsilon > 0$ , there exists a local CP instrument and corresponding measurement process on the global algebra  $\mathfrak{K}$  which approximates  $\mathcal{I}$  to within error  $\epsilon$ .*

# Interpretive Questions

- 1.) What is the relationship between the Okamura-Ozawa and the Fewster-Verch frameworks?

# Interpretive Questions

- 1.) What is the relationship between the Okamura-Ozawa and the Fewster-Verch frameworks?
  - ▶ Converse projects? FV probe limitations?

# Interpretive Questions

- 1.) What is the relationship between the Okamura-Ozawa and the Fewster-Verch frameworks?
  - ▶ Converse projects? FV probe limitations?
  - ▶ State change localization? Epistemic v. Ontic?

# Interpretive Questions

- 1.) What is the relationship between the Okamura-Ozawa and the Fewster-Verch frameworks?
  - ▶ Converse projects? FV probe limitations?
  - ▶ State change localization? Epistemic v. Ontic?
  - ▶ Impossible measurements? Primitive causality v. split property?

# Interpretive Questions

- 1.) What is the relationship between the Okamura-Ozawa and the Fewster-Verch frameworks?
  - ▶ Converse projects? FV probe limitations?
  - ▶ State change localization? Epistemic v. Ontic?
  - ▶ Impossible measurements? Primitive causality v. split property?
- 2.) How should we frame the relativistic measurement problem?
  - ▶ Hope for conservatism: just like QM?

# Interpretive Questions

- 1.) What is the relationship between the Okamura-Ozawa and the Fewster-Verch frameworks?
  - ▶ Converse projects? FV probe limitations?
  - ▶ State change localization? Epistemic v. Ontic?
  - ▶ Impossible measurements? Primitive causality v. split property?
- 2.) How should we frame the relativistic measurement problem?
  - ▶ Hope for conservatism: just like QM?
  - ▶ State change? Local quantities v. operations? Local superposition? (Buchholz & Størmer 2015)

# Interpretive Questions

- 1.) What is the relationship between the Okamura-Ozawa and the Fewster-Verch frameworks?
  - ▶ Converse projects? FV probe limitations?
  - ▶ State change localization? Epistemic v. Ontic?
  - ▶ Impossible measurements? Primitive causality v. split property?
- 2.) How should we frame the relativistic measurement problem?
  - ▶ Hope for conservatism: just like QM?
  - ▶ State change? Local quantities v. operations? Local superposition? (Buchholz & Størmer 2015)
  - ▶ Repeatability loophole?

# Interpretive Questions

- 1.) What is the relationship between the Okamura-Ozawa and the Fewster-Verch frameworks?
  - ▶ Converse projects? FV probe limitations?
  - ▶ State change localization? Epistemic v. Ontic?
  - ▶ Impossible measurements? Primitive causality v. split property?
- 2.) How should we frame the relativistic measurement problem?
  - ▶ Hope for conservatism: just like QM?
  - ▶ State change? Local quantities v. operations? Local superposition? (Buchholz & Størmer 2015)
  - ▶ Repeatability loophole?
- 3.) How do we measure local counterparts of quasilocal and global observables?

# Interpretive Questions

- 1.) What is the relationship between the Okamura-Ozawa and the Fewster-Verch frameworks?
  - ▶ Converse projects? FV probe limitations?
  - ▶ State change localization? Epistemic v. Ontic?
  - ▶ Impossible measurements? Primitive causality v. split property?
- 2.) How should we frame the relativistic measurement problem?
  - ▶ Hope for conservatism: just like QM?
  - ▶ State change? Local quantities v. operations? Local superposition? (Buchholz & Størmer 2015)
  - ▶ Repeatability loophole?
- 3.) How do we measure local counterparts of quasilocal and global observables?
  - ▶ Local aspects of superselection rules? Universal localizing map? (Doplicher et al. 1982-1987)

# References

- Arveson (1967), Analyticity in Operator Algebras
- Bostelmann, Fewster, Ruep (2021), Impossible Measurements Require Impossible Apparatus
- Buchholz, Doplicher, Longo (1985), On Noether's Theorem in QFT
- Buchholz, Størmer (2015), Superposition, Transition Probabilities, and Primitive Observables in Infinite Quantum Systems
- D'Antoni, Doplicher, Fredenhagen, Longo (1987), Convergence of Local Charges and Continuity Properties of  $W^*$ -Inclusions
- Davies, Lewis (1970), An Operational Approach to Quantum Probability
- Doplicher (1982), Local Aspects of Superselection Rules
- Doplicher, Longo (1983), Local Aspects of Superselection Rules II
- Doplicher, Longo (1984), Standard and Split Inclusions of von Neumann Algebras
- Fewster, Verch (2020), Quantum Fields and Local Measurements
- Kraus (1971), General State Changes in Quantum Theory
- Lüders (1951), Concerning the State-Change Due to the Measurement Process
- Nakamura, Umegaki (1962), On von Neumann's Theory of Quantum Measurement and Statistics
- Okamura, Ozawa (2016), Measurement Theory in Local Quantum Physics
- Ozawa (1984), Quantum Measuring Process of Continuous Observables
- von Neumann (1932), *Mathematical Foundations of Quantum Mechanics*