Facets of relativistic locality

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Main message

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- The categorial approach to QFT provides a flexible and general framework to formulate relativistic locality conditions.
- One should be **very (?)** cautiously optimistic about whether quantum field theory satisfies the relativistic locality conditions.

Outline

1. Relativistic locality conditions informally
2. Categorial formulation of quantum field theory
3. Relativistic Locality conditions in categorial QFT
4. Why the cautious optimism?

Relativistic locality as a set of features – informally

- **Spatio-temporal Locality**: Physical systems are localized explicitly in spacetime regions.
- **Causal Locality**: The observational-operational properties of the physical systems localized in spacetime regions are in harmony with the causal relations between the spacetime regions:

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  - **Causal Locality – Dynamic**: The dynamical evolution of a system in a region determines system in the region's causal closure.

Comments on the semi-formal notion of relativistic locality

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- The Causal Locality condition Explanation might not be conceptually independent of the Causal Locality condition Dynamic.

Categorial relativistic quantum field theory

Quantum field theory ... is a covariant functor ... in the ... fundamental and physical sense of implementing the principles of locality and general covariance...

R. Brunetti, K. Fredenhagen, R. Verch: "The generally covariant locality principle. A new paradigm for local quantum field theory"

Communications in Mathematical Physics **237** *(2003) 61-78*

Categorial formulation of quantum field theory

Idea (Fredenhagen, Brunetti and Verch 2003)

Formulate locality and general covariance in terms of a functor $\mathcal{F}$ between two categories:

- $(\mathfrak{Man}, hom_{\mathfrak{Man}})$
  category of spacetimes
  with isometric embeddings of spacetimes as morphisms
- $(\mathfrak{Alg}, hom_{\mathfrak{Alg}})$
  category of C^*-algebras
  with injective C^*-algebra homomorphisms as morphisms

Locally covariant categorial quantum field theory

Definition

A locally covariant quantum field theory is a covariant functor $\mathcal{F}$ between the categories $(\mathfrak{Man}, hom_{\mathfrak{Man}})$ and $(\mathfrak{Alg}, hom_{\mathfrak{Alg}})$

$$\begin{array}{ccc} (M,g) & \xrightarrow{\quad\psi\quad} & (M',g') \\ \mathcal{F}\big\downarrow & & \big\downarrow\mathcal{F} \\ \mathcal{F}(M,g) & \xrightarrow{\quad\mathcal{F}(\psi)\quad} & \mathcal{F}(M',g') \end{array}$$

$$\begin{aligned} \mathcal{F}(\psi_1 \circ \psi_2) &= \mathcal{F}(\psi_1) \circ \mathcal{F}(\psi_2) \\ \mathcal{F}(id_{\mathfrak{Man}}) &= id_{\mathfrak{Alg}} \end{aligned}$$

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Relativistic locality conditions
to be formulated as postulates on the functor $\mathcal{F}$.

Relativistic locality conditions postulated for the functor

- Spatiotemporal locality is expressed by the functor itself:
  - The spatiotemporal locality of system with observables $\mathcal{F}(M, g)$ is M.

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operation

$\|$

completely positive unit preserving map
between C^*-algebras

Locally covariant categorial quantum field theory – Einstein Causality

Definition

The functor $\mathcal{F}\colon (\mathfrak{Man}, hom_{\mathfrak{Man}}) \to (\mathfrak{Alg}, hom_{\mathfrak{Alg}})$ is called

- (Einstein) Causal if

$$\Big[\mathcal{F}(\psi_1)\Big(\mathcal{F}(M_1, g_1)\Big), \mathcal{F}(\psi_2)\Big(\mathcal{F}(M_2, g_2)\Big)\Big]_{-}^{\mathcal{F}(M,g)} = \{0\}$$

  whenever

$$\begin{aligned} \psi_1 &: (M_1, g_1) \to (M, g) \\ \psi_2 &: (M_2, g_2) \to (M, g) \end{aligned}$$

  and $\psi_1(M_1)$ and $\psi_2(M_2)$ are spacelike in M

Locally covariant categorial quantum field theory – Time slice axiom

Definition

If (M, g) and (M', g') and

$$\psi \colon (M, g) \to (M', g')$$

are such that $\psi(M, g)$ contains a Cauchy surface for (M', g') then

$$\mathcal{F}(\psi)\mathcal{F}(M, g) = \mathcal{F}(M', g')$$

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Notational simplification:

In what follows g is left out from $\mathcal{F}(M, g)$:

$$\mathcal{F}(M, g) = \mathcal{F}(M)$$

Operational independence – informally

Operational independence is a special instance of

subsystem independence as co-possibility:

Assume that S_1 and S_2 are two subsystems of a larger system S

Then S_1 and S_2 are independent if

- Anything which is possible in principle for S_1 as a system
  in its own right
  and
- anything which is possible in principle for S_2 as a system
  in its own right
  are
- also jointly possible in principle for the pair (S_1, S_2) viewed as subsystems of S

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Operational independence:
Any two operations on subsystems S_1, S_2
are jointly realizable
as an operation on the system S

Operational independence in QFT – semi-formally

$M_1, M_2 \subset M$ spacetime regions
M_1 spacelie separated from M_2
$\mathcal{A}(M_1), \mathcal{A}(M_2) \subset \mathcal{A}(M)$
algebras of observables in the regions

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Operational independence:

Any two operations
T_1 on $\mathcal{A}(M_1)$ and T_2 on $\mathcal{A}(M_2)$
have a joint extension
to an operation T on $\mathcal{A}(M)$

Content of operational independence – state change view

Operations express interactions with the system (e.g. measurements) that change the system's state

$\Downarrow$

Operational independence

$\|$

Any two state transitions of the form

$$\phi_1 \mapsto \phi_1 \circ T_1 \qquad\qquad \phi_2 \mapsto \phi_2 \circ T_2$$

are co-possible:
realizable as a transition of a single state of S:

$$\phi \mapsto \phi \circ T$$

ψ-extension of operations

Definition

Given

$$\begin{array}{ccl} \psi & : & (M,g) \to (M',g') \\ T & & \text{operation on } \mathcal{F}(M) \\ T' & & \text{operation on } \mathcal{F}(M') \end{array}$$

The operation T' is called the ψ-extension of T if the following diagram is commutative:

$$\begin{array}{ccc} \mathcal{F}(M) & \xrightarrow{\mathcal{F}(\psi)} & \mathcal{F}(M') \\ T \downarrow & & \downarrow T' \\ \mathcal{F}(M) & \xrightarrow{\mathcal{F}(\psi)} & \mathcal{F}(M') \end{array}$$

Causal Locality – operational independence, formally

Definition

The functor $\mathcal{F}\colon (\mathfrak{Man}, hom_{\mathfrak{Man}}) \rightarrow (\mathfrak{Alg}, hom_{\mathfrak{Alg}})$ is said to satisfy the operational independence condition

if whenever

$$\psi_1\colon (M_1, g_1) \rightarrow (M, g)$$
$$\psi_2\colon (M_2, g_2) \rightarrow (M, g)$$

and $\psi_1(M_1)$ and $\psi_2(M_2)$ are spacelike in M

then

for any operation T_1 on $\mathcal{F}(M_1)$

and

for any operation T_2 on $\mathcal{F}(M_2)$

there is

an operation T on $\mathcal{F}(M)$

which is a ψ_1-extension of T_1 and a ψ_2-extension of T_2

Operational separability – informally

Operational separability is a no superluminal signaling condition:

Assume T_1 represents an operation performed on a physical system localized in spacetime region M_1.

Then, operational separability says:

If this operation T_1 can be carried out as an operation on a larger system localized in $M \supset M_1$, then it can be carried out on the larger system localized in M also in such a way that the operation leaves intact any physical system localized in another part $M_2 \subset M$ that is spacelike from M_1.

Operational separability – semi-formally

$M_1, M_2 \subset M$ spacetime regions
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Operational separability:

If T_1 is an operation on $\mathcal{A}(M_1)$
and there exists an extension T of T_1 to $\mathcal{A}(M)$
then there exists an extension T' of T_1 to $\mathcal{A}(M)$
that is the identity operation on $\mathcal{A}(M_2)$
(and the same holds if indices $1 \leftrightarrow 2$ are interchanged)

Causal Locality – operational separability

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then

for any operation T_1 on $\mathcal{F}(M_1)$

there is

an operation T on $\mathcal{F}(M)$

which is a ψ_1-extension of T_1 and is the identity operation on $\mathcal{F}(M_2)$

similarly with a T_2 on $\mathcal{F}(M_2)$ and identity on $\mathcal{F}(M_1)$

Comment

Einstein causality

$\Downarrow$

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with respect to spatiotemporally local
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Einstein causality $\Rightarrow$ No-signaling
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A general operation on a general C^*-algebra

is not Kraus representable

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with respect to spatiotemporally local
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But
A general operation on a general C^*-algebra
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Einstein causality $\not\Rightarrow$ No-signaling
with respect to general operations

Operational independence and operational separability

- It is obvious that

$$\Big[\text{ operational independence }\Big] \Rightarrow \Big[\text{ operational separability }\Big]$$

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- Converse?

$$\Big[\text{ operational independence }\Big] \overset{?}{\not\Leftarrow} \Big[\text{ operational separability }\Big]$$

Content of Causal Locality – Explanation – informally

The Causal Dependence – Explanation condition requires that a (possibly operator valued) correlation predicted by an operation between operators lying in algebras pertaining to spaceike separated spacetime regions is "explainable" by an operation on a local algebra associated with a region lying in the common causal past of the regions containing the correlated operators.

Explainable: manipulating (conditionalizing, i.e. composing) the correlated operation with an operation in the causal past of the correlated operators makes the correlation disappear.

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The Causal locality – Explanation condition is the formulation of the Common Cause Principle in terms of categorial QFT.

Correlated operations

Definition

Given

$$\psi_1 \quad : \quad (M_1, g_1) \to (M, g)$$
$$\psi_2 \quad : \quad (M_2, g_2) \to (M, g)$$

with $\psi_1(M_1)$ and $\psi_2(M_2)$ spacelike in M

the operation

$$T \text{ on } \mathcal{F}(M)$$

is said to be (ψ_1, ψ_2)-correlated if for some $X \in \mathcal{F}(M_1)$, $Y \in \mathcal{F}(M_2)$

$$T(\mathcal{F}(\psi_1)(X)\mathcal{F}(\psi_2)(Y)) \neq T(\mathcal{F}(\psi_1)(X))T(\mathcal{F}(\psi_2)(Y))$$

Causal Locality – Explanation

Definition

The functor $\mathcal{F}\colon (\mathfrak{Man}, hom_{\mathfrak{Man}}) \to (\mathfrak{Alg}, hom_{\mathfrak{Alg}})$ is said to satisfy the

Causal locality – Explanation condition

if whenever some operation T on $\mathcal{F}(M))$ is (ψ_1, ψ_2)-correlated (on $X \in \mathcal{F}(M_1)$, $Y \in \mathcal{F}(M_2)$) then there exist

$$(M_0, g_0) \text{ and } \psi_0\colon (M_0, g_0) \to (M, g)$$

with

$$\psi_0(M_0) \text{ in the causal past of both } \psi_1(M_1) \text{ and } \psi_2(M_2)$$

and an operation

$$T_0 \text{ on } \mathcal{F}(M_0)$$

which screens off the correlation:
T_0 has a ψ_0-extension $T_0^{\psi_0}$ from $\mathcal{F}(M_0)$ to $\mathcal{F}(M)$ for which we have

$$(T \circ T_0^{\psi_0})(\mathcal{F}(\psi_1)(X)\mathcal{F}(\psi_2)(Y)) = (T \circ T_0^{\psi_0})(\mathcal{F}(\psi_1)(X))(T \circ T_0^{\psi_0})(\mathcal{F}(\psi_2)(Y))$$

Causally local functor and Main claim

Definition

A covariant functor

$$\mathcal{F}\colon (\mathfrak{Man}, hom_{\mathfrak{Man}}) \to (\mathfrak{Alg}, hom_{\mathfrak{Alg}})$$

which satisfies

- Einstein locality
- Time-slice axiom
- Operational independence
- Operational explanation

is called a causally local functor.

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A causally local functor captures, in terms of category theory, the intuition about what it means for a quantum theory to be relativistically local: A particular quantum field theory is in compliance with relativistic locality if it can be formulated in terms of a covariant causally local functor.

Why very cautious optimism?

- The Haag-Kastler algebraic quantum field theory (AQFT) can be recovered as a particular case of categorial quantum field theory.
- Operational independence does hold in AQFT for systems localized in strictly spacelike separated double cone regions.
- Correlations in AQFT predicted in AQFT by states between spacelike separated observables can be screened off by operations localized in the union (rather than intersection) of the backward light cones of the spacelike separated regions containing the correlated observables.

Closing comments

- One could consider special subclasses of operations and require the operational causal locality conditions to hold for the operations in that subclass.
- One can in principle replace the operations with other morphisms in the class of C^*-algebras.
- $\Rightarrow$ Qualitatively different relativistic locality conditions – the morphism class is a variable in the categorial concept of relativistic locality.

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- $\Rightarrow$ Qualitatively different relativistic locality conditions – the morphism class is a variable in the categorial concept of relativistic locality.

Relativistic locality is multifaceted indeed!

References

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