# State updates and useful qubits in relativistic quantum information

J. Polo-Gómez, T. Rick Perche, E. M-M

arXiv:2506.18906

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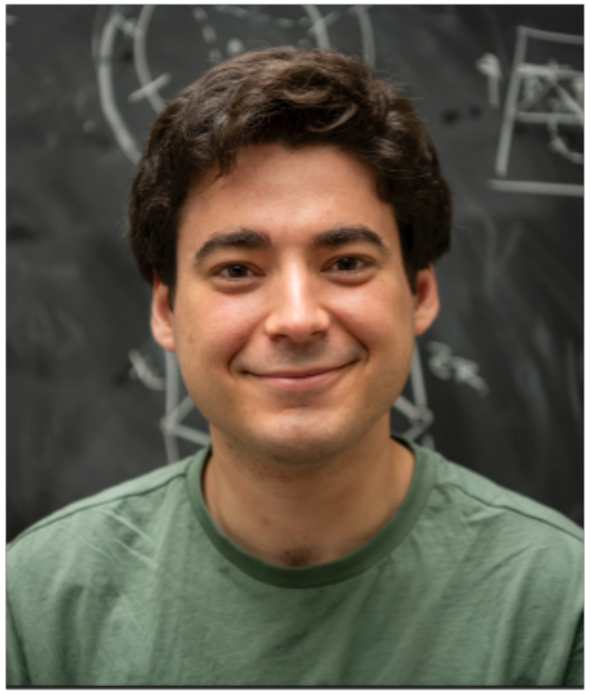
barrio-RQI.org

Eduardo Martín-Martínez
Department of Applied Mathematics (University of Waterloo)
Inst. for Quantum Computing
Perimeter Institute for Theoretical Physics

# Will talk about work in collaboration with



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# Measurement Problem in QFT

Quantum Temporal Probabilities (QTP) method

C. Anastopoulos, B. Hu, K. Savvidou
J. Phys.: Conf. Ser. 2533 012004 (2023)
Annals of Physics 450, 169239 (2023)

Positive formalism

R. Oeckl, Advances in Theoretical and Mathematical Physics 23, 2, 437 (2019)
R. Oeckl, A. Zampeli, arXiv:2505.10968 (2025)

Fewster-Verch framework

C. J. Fewster, R. Verch
Commun. Math. Phys., 378(2):851–889, 2020.
C. J. Fewster (2019), arXiv:1904.06944
C. J. Fewster, R. Verch arXiv:2304.13356
H. Bostelmann, C. J. Fewster, and M. H. Ruep, Phys. Rev. D 103, 025017 (2021)
C. J. Fewster, I. Jubb, and M. H. Ruep (2022), arXiv:2203.09529

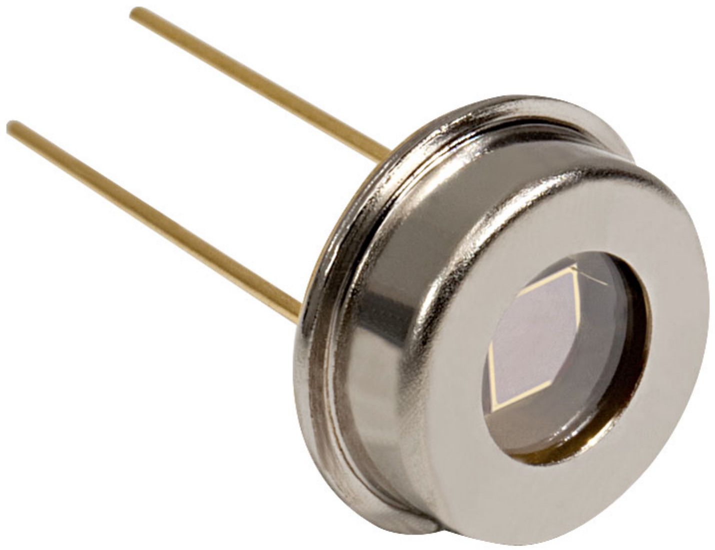
Detector based measurement theory

J. Polo-Gómez, L. J. Garay, E. M-M, Phys. Rev. D 105, 065003 (2022)
N. Pranzini, E. Keski-Vakkuri, arXiv:2310.06596

For a great overview:

M. Papageorgiou, D. Fraser
Eliminating the 'Impossible': Recent Progress on Local Measurement Theory for Quantum Field Theory.
*Found. Phys.* **54**, 26 (2024)

# Several approaches



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# (Essentially) Covariant particle detectors

Causality and covariant formulation of the model:

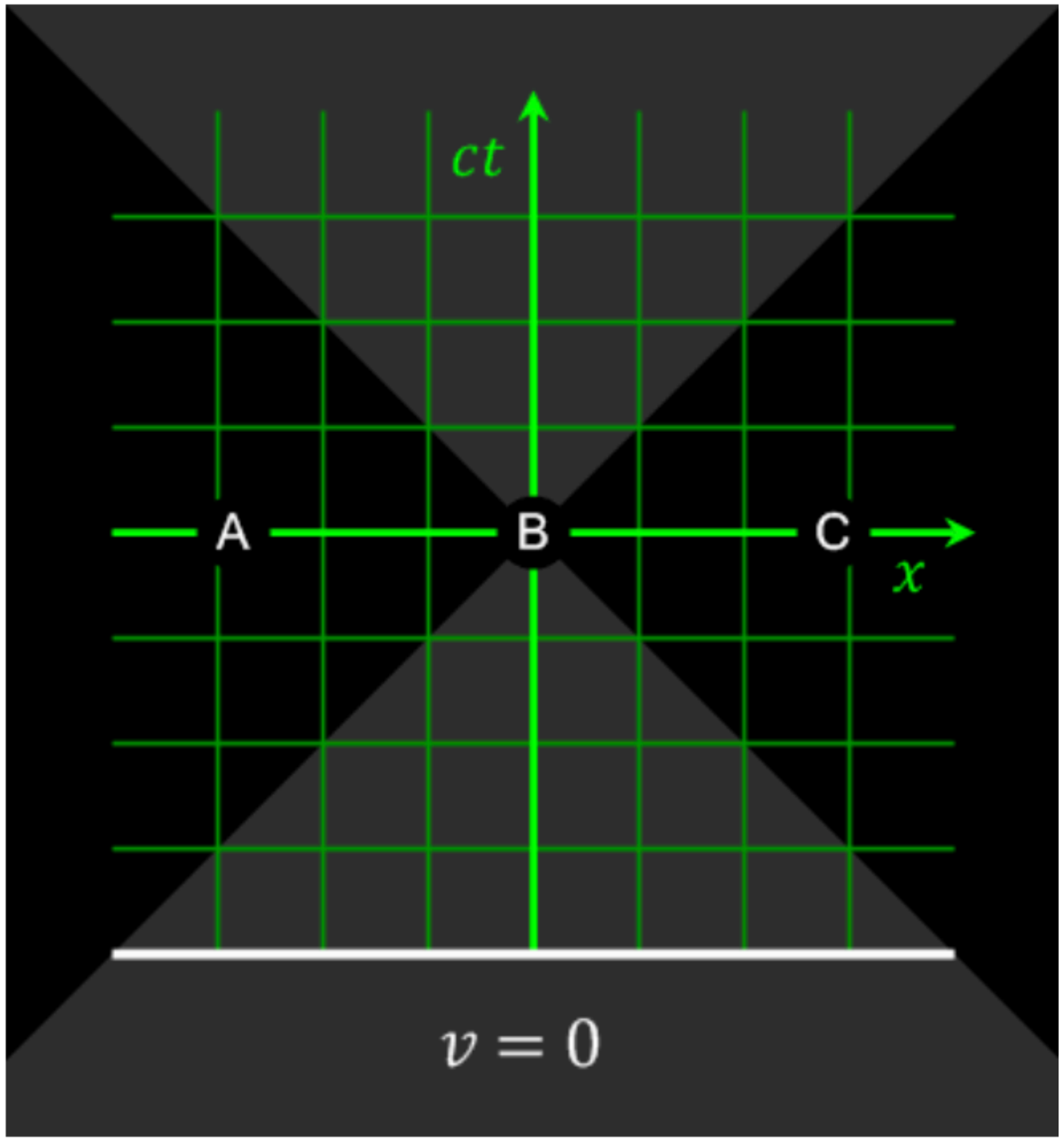
E. M-M, Phys. Rev. D 92, 104019 (2015)
E. M-M, P. Rodriguez-Lopez. Phys. Rev. D 97, 105026 (2018)
E. M-M., T. Rick Perche, Bruno, S. L. Torres. Phys. Rev. D 101, 045017 (2020)
E. M-M., T. Rick Perche, Bruno, S. L. Torres. Phys. Rev. D 103, 025007 (2021)
J. de Ramón, M. Papageorgiou, E. M-M. Phys. Rev. D 103, 085002 (2021)
T. Rick Perche. Phys. Rev. D 106, 025018 (2022)
J. de Ramón, M. Papageorgiou, E. M-M. Phys. Rev. D 108, 045015 (2023)

Emergence of UDW detectors from a fully relativistic quantum field theory relating them to FV:

T. Rick Perche, J. Polo-Gómez, B. de S. L. Torres, E. M-M, Phys. Rev. D 109, 045013 (2024)
T. Rick Perche, J. Polo-Gómez, B. de S. L. Torres, E. M-M, Phys. Rev. D 109, 045018 (2024)

# State updates in spacetime

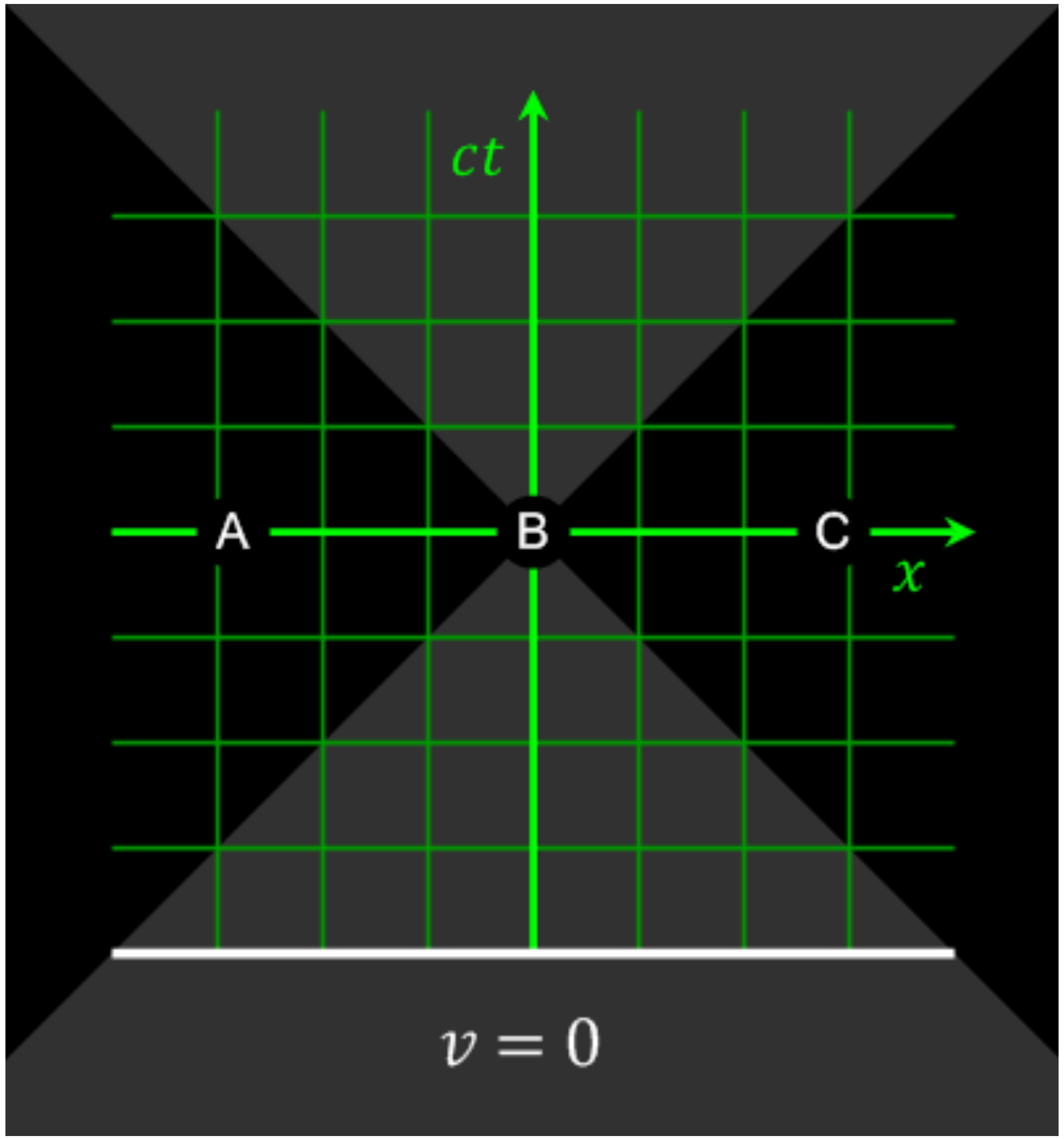
In non-relativistic quantum mechanics with a notion of absolute time a measurement happens at some time and the whole state is updated at the same time




Consistency with relativity is usually justified by the no-signalling theorem and an external causal structure

# State updates in spacetime

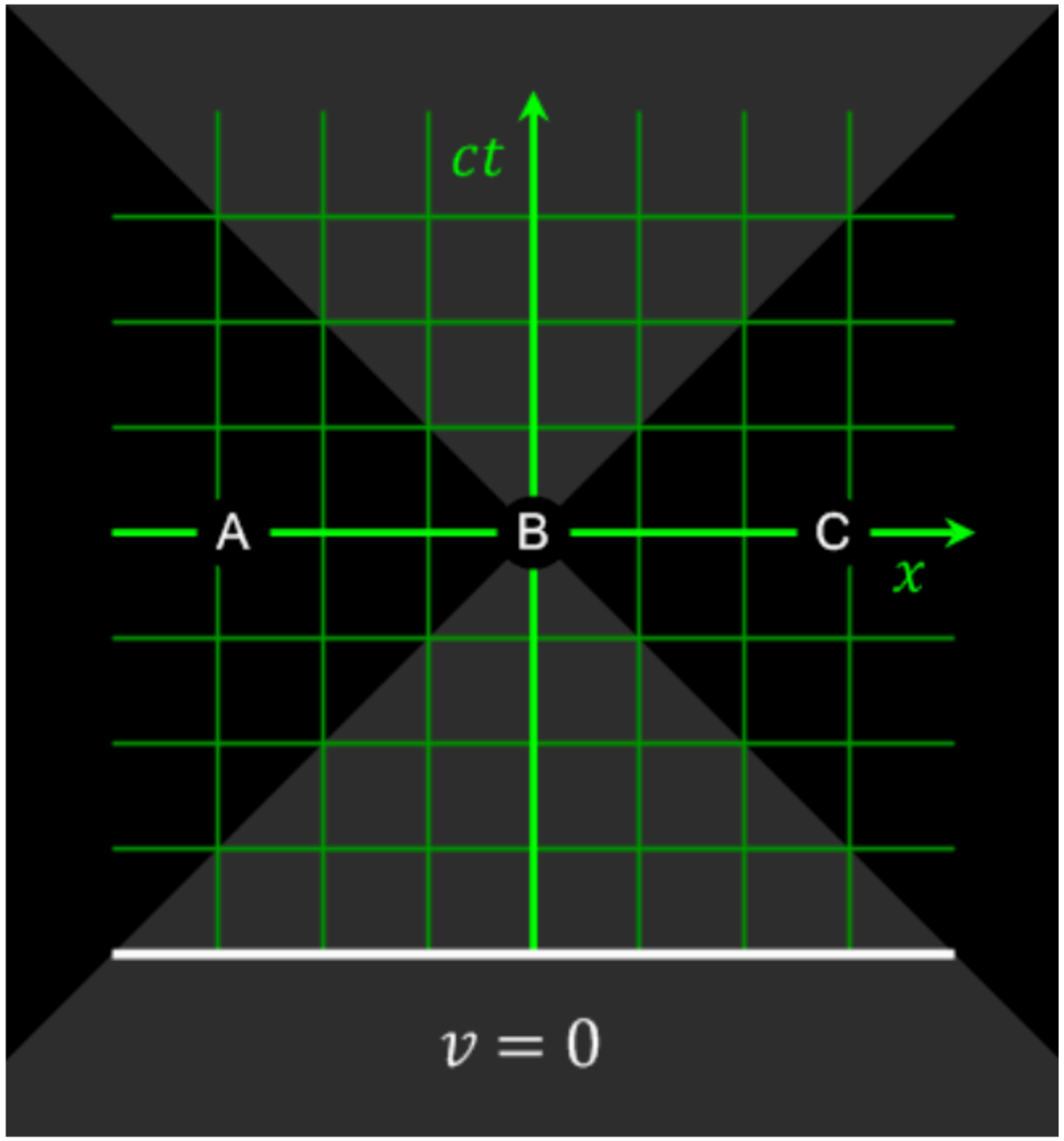
In relativistic settings there is no notion of absolute time




Is there a relativistic notion of state update that we can build?
Where/When does it happen?

# State updates in spacetime

In relativistic settings there is no notion of absolute time

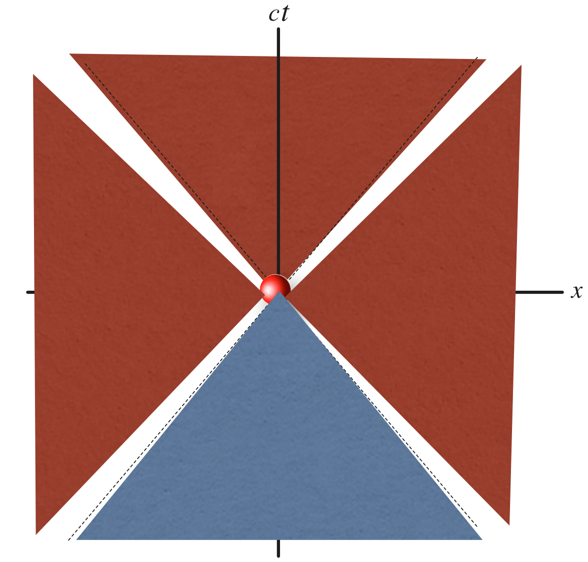



Can a state-update rule be both relativistically covariant and fully predictive

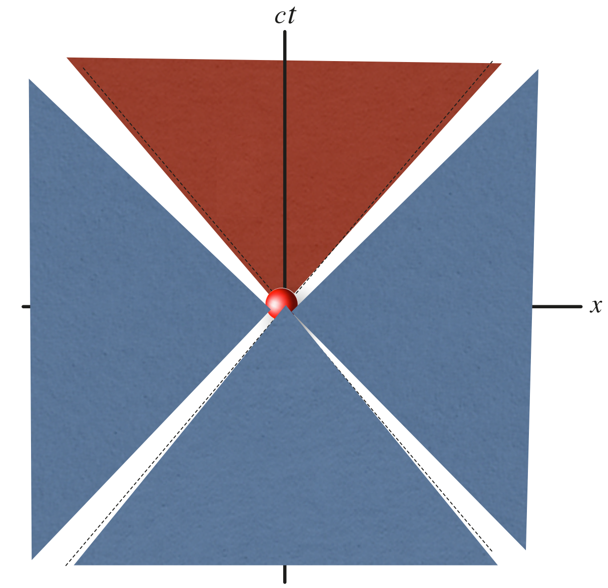
# State updates in spacetime

If we insist on covariance, there are really only two canonical hypersurfaces attached to an event: its past lightcone and its future lightcone.

- Update along the past lightcone of the measurement [1]
  - This is the proposed update by Hellwig and Kraus




- Update along the future lightcone of the measurement
  - May intuitively appear to respect causal propagation of information





[1] K. E. Hellwig and K. Kraus, Formal Description of Measurements in Local Quantum Field Theory, Phys. Rev. D 1, 566 (1970).

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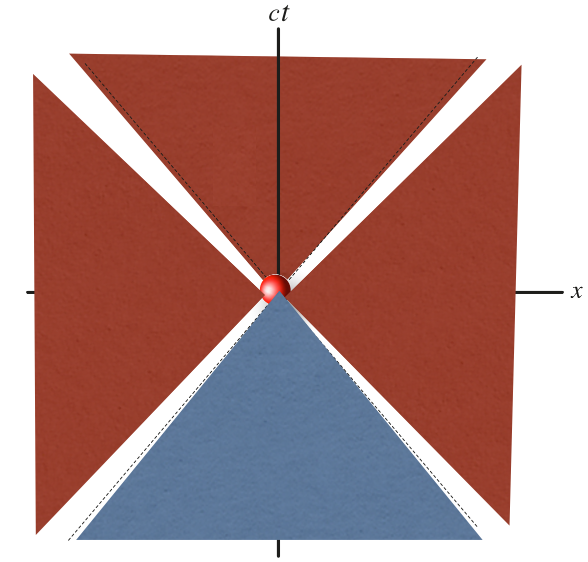
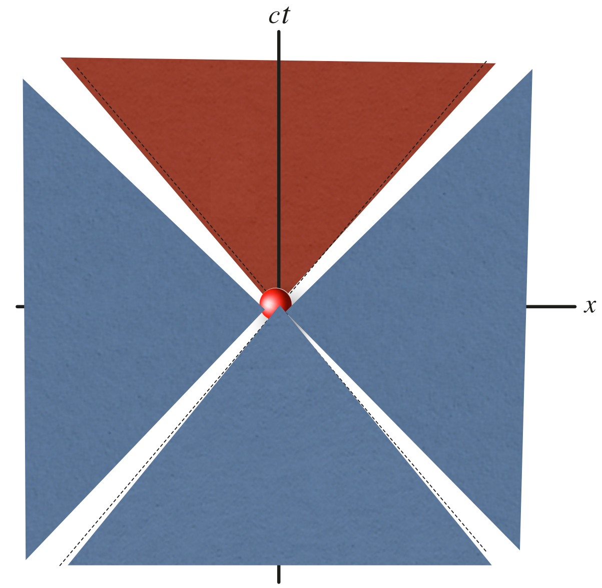



Let’s test them

- Update along the future lightcone of the measurement
  - May intuitively appear to respect causal propagation of information

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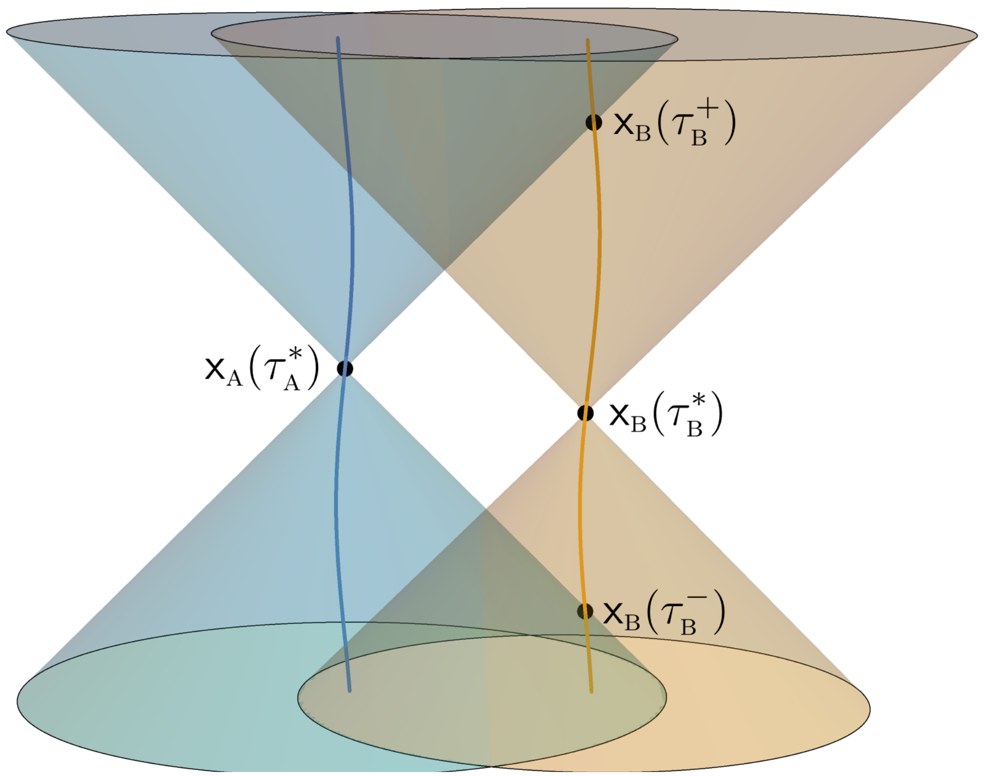
- Hellwig & Kraus: Measurements alter the state everywhere outside causal past of the measurement




Criticism of Aharonov and Albert:

Y. Aharonov and D. Z. Albert, Can we make sense out of the measurement process in relativistic quantum mechanics?, Phys. Rev. D 24, 359 (1981)

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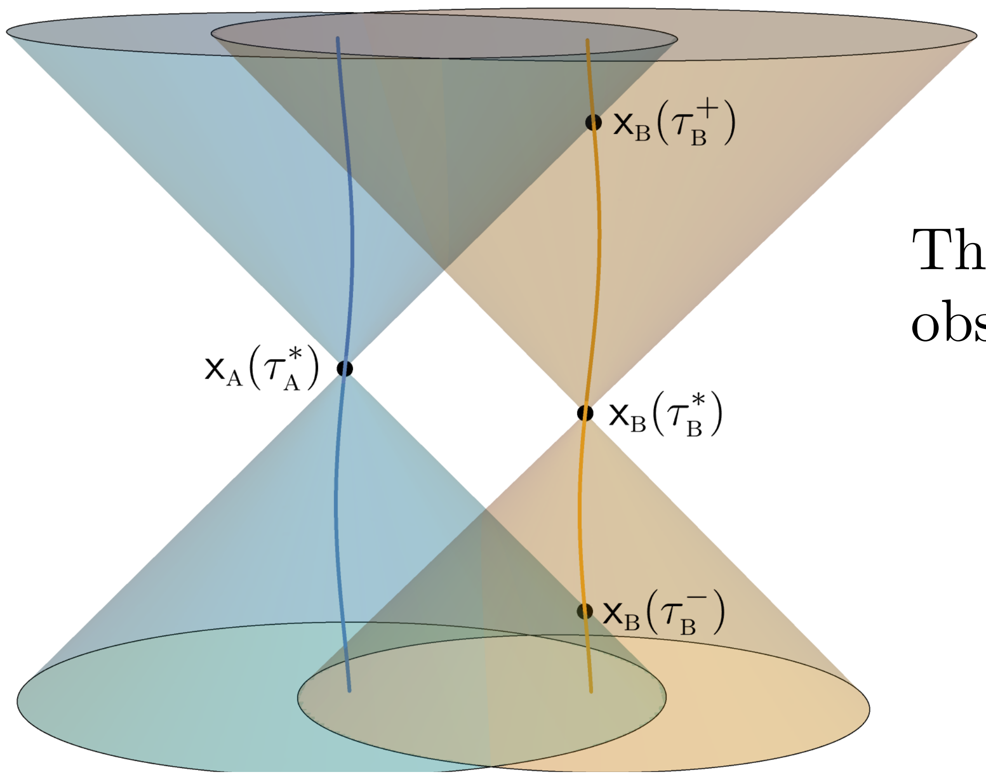


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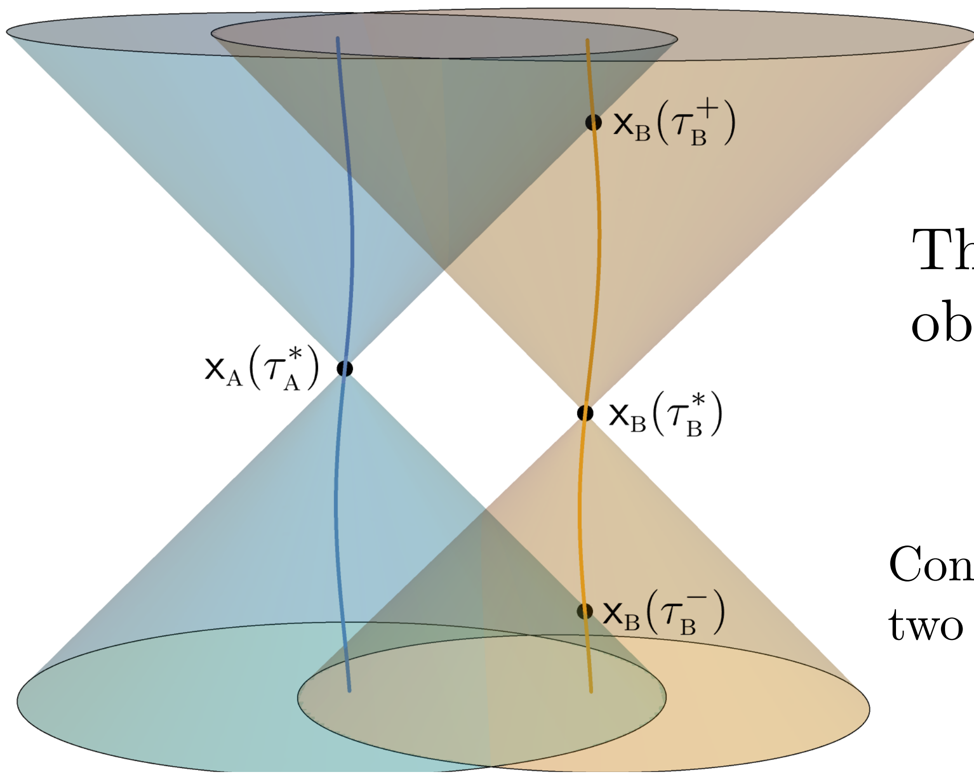


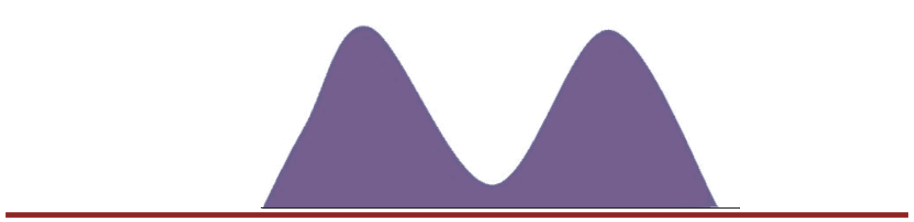

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## Charge conservation problem

Consider an electron initially spatially delocalized, equally distributed between two distant regions, A and B.


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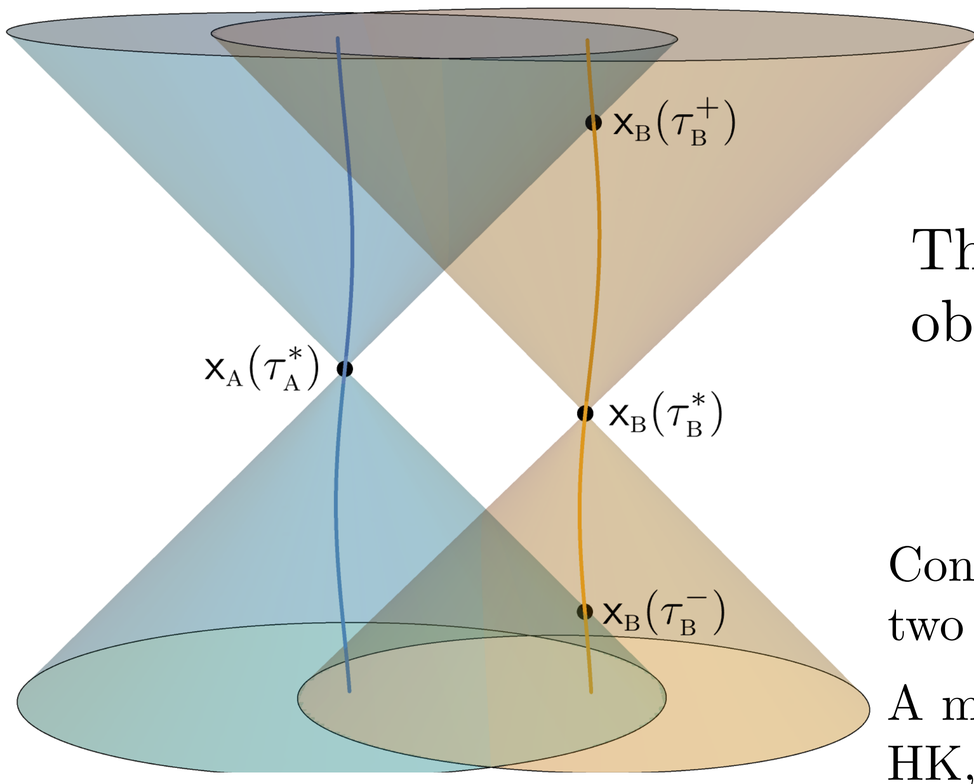


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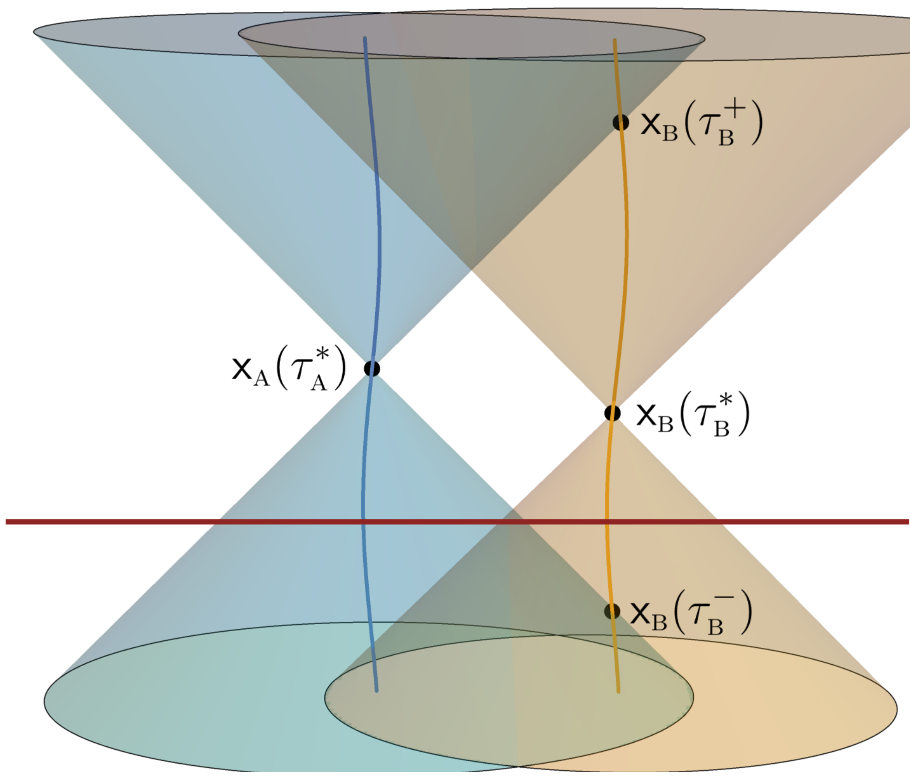


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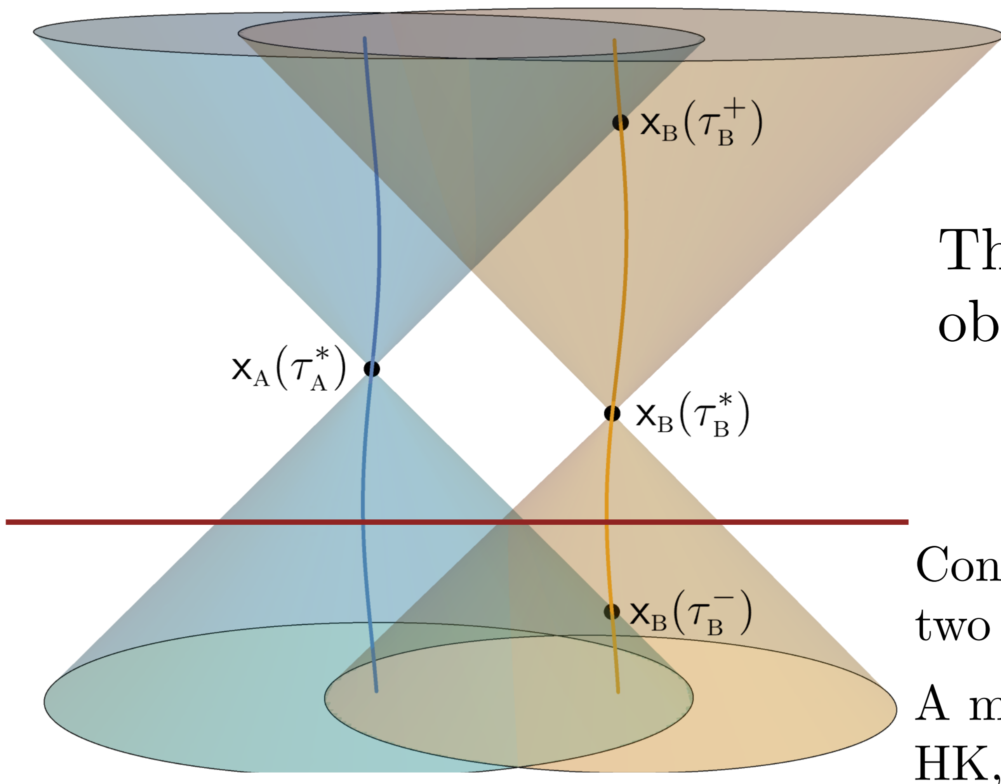


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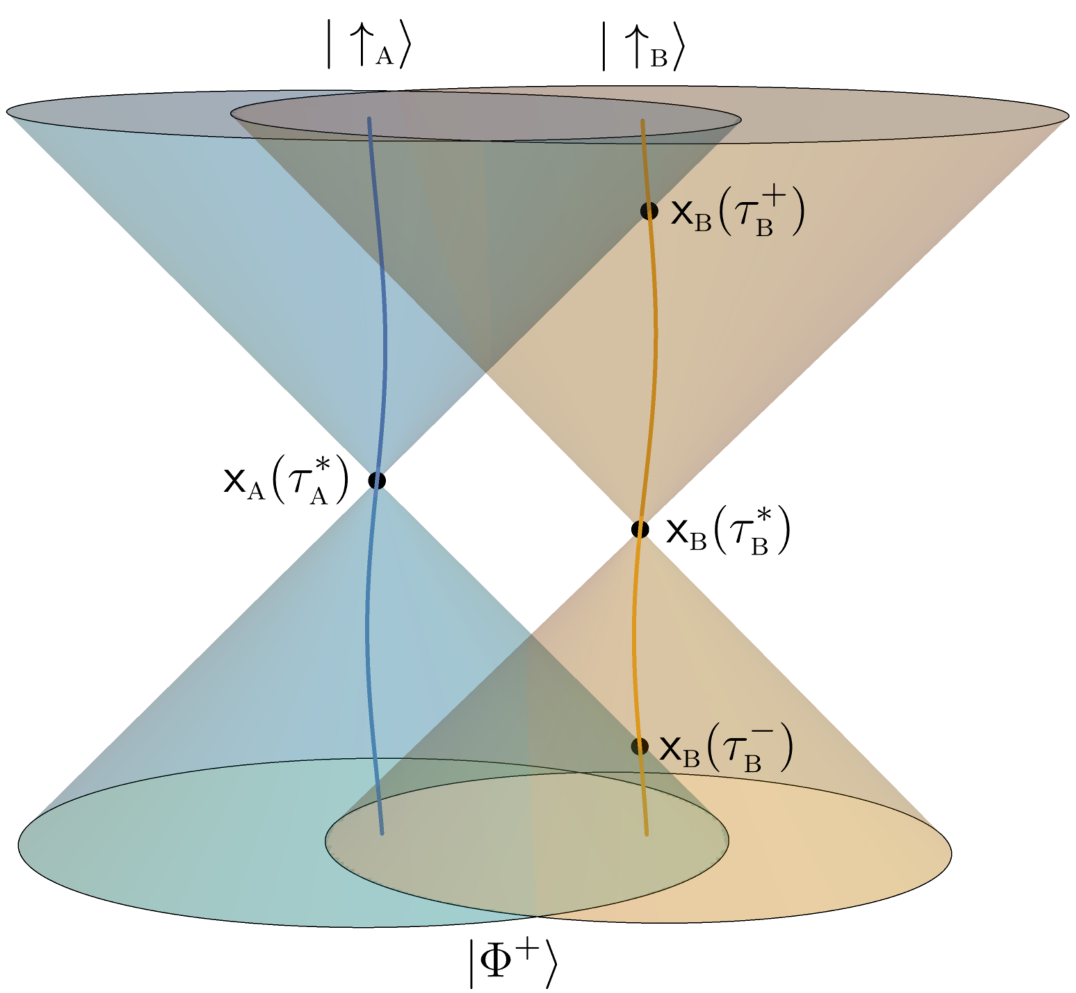
Integrating the local charge densities across the slice, one gets less than the total electron charge.


Y. Aharonov and D. Z. Albert, Can we make sense out of the measurement process in relativistic quantum mechanics?, Phys. Rev. D 24, 359 (1981)

# Update along the Past Lightcone?

- Consider an EPR pair

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}\left(|{\uparrow_{\rm A}}\rangle\,|{\uparrow_{\rm B}}\rangle + |{\downarrow_{\rm A}}\rangle\,|{\downarrow_{\rm B}}\rangle\right) \in \mathcal{H}_{\rm A} \otimes \mathcal{H}_{\rm B}$$



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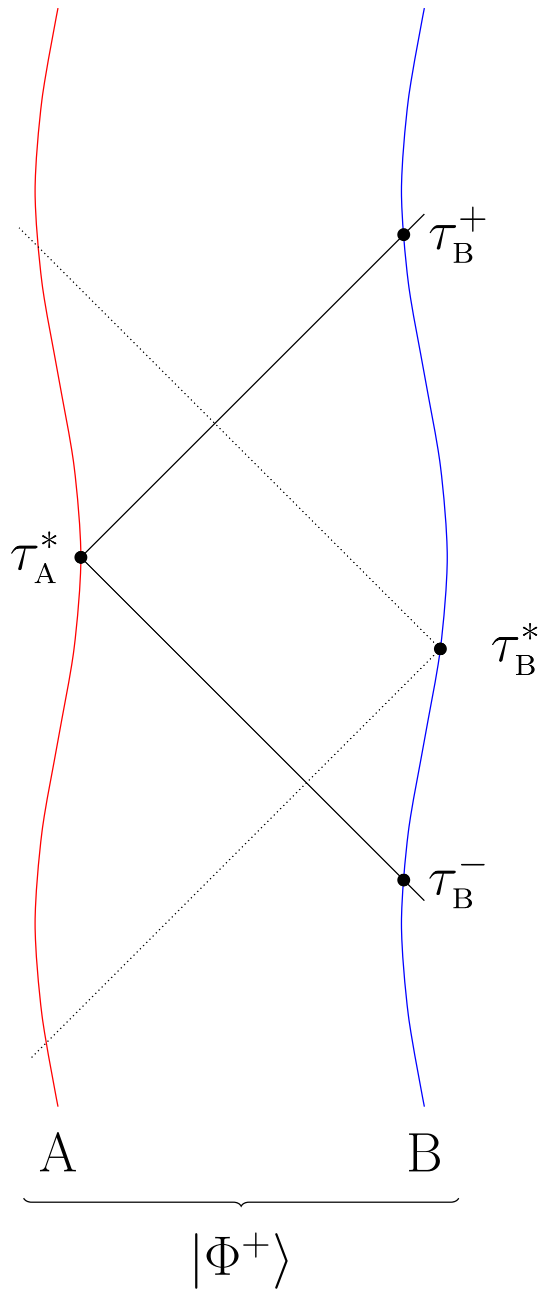
- Effective retrocausal chain?



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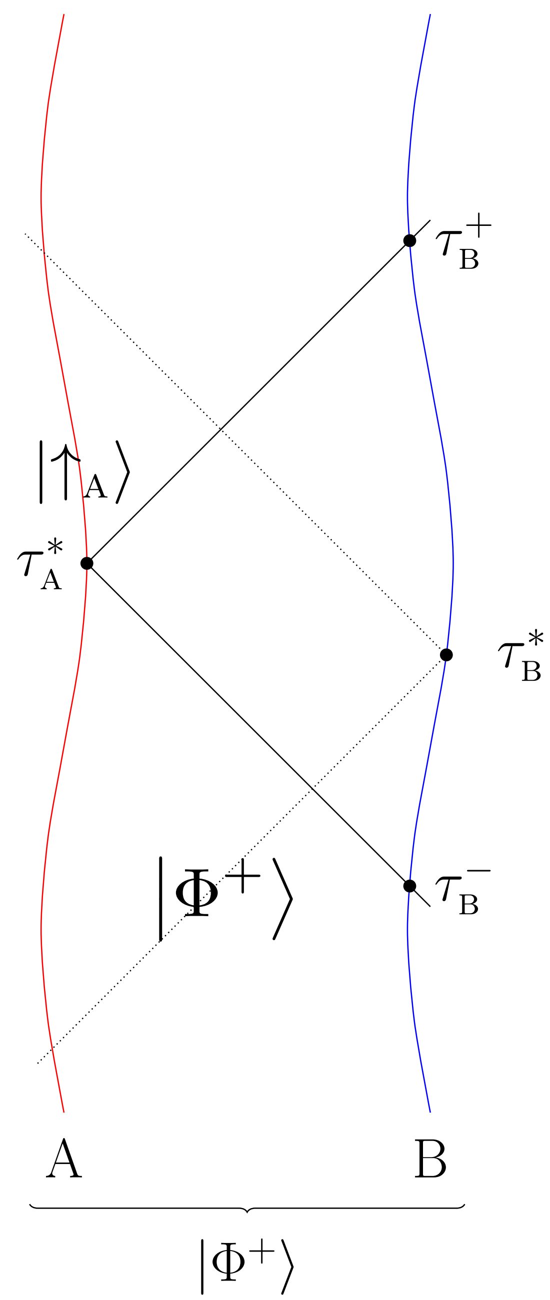

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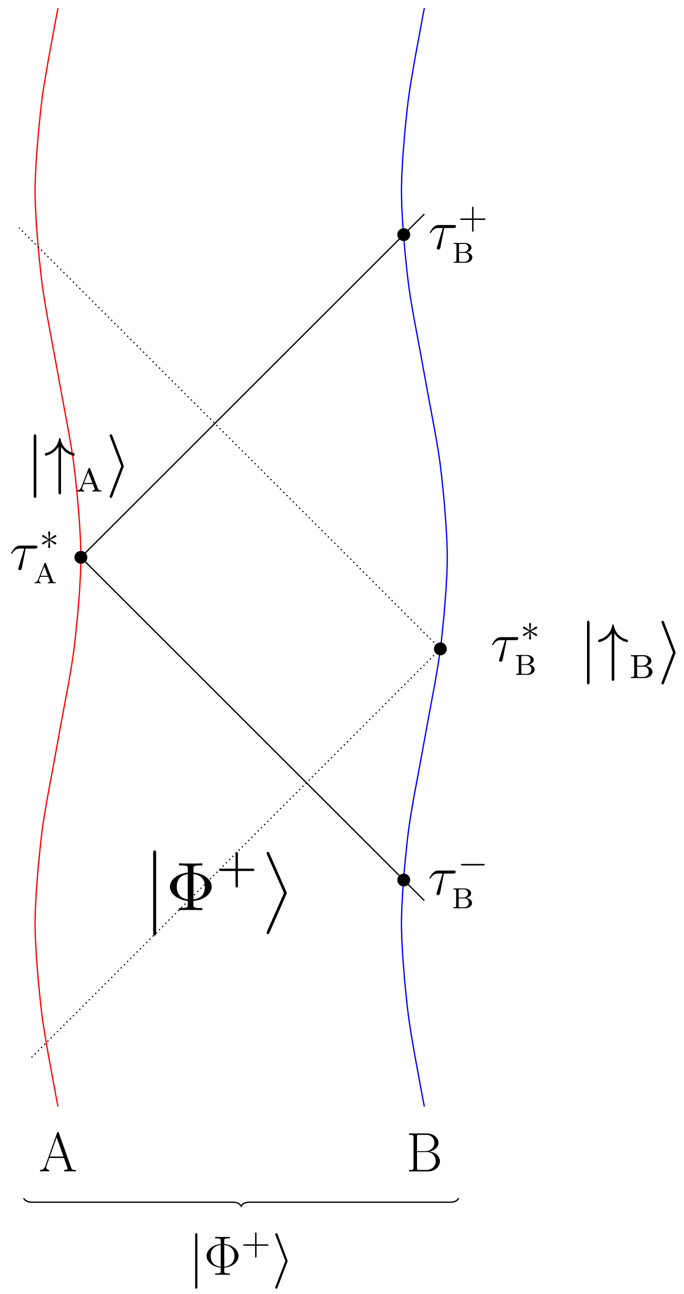

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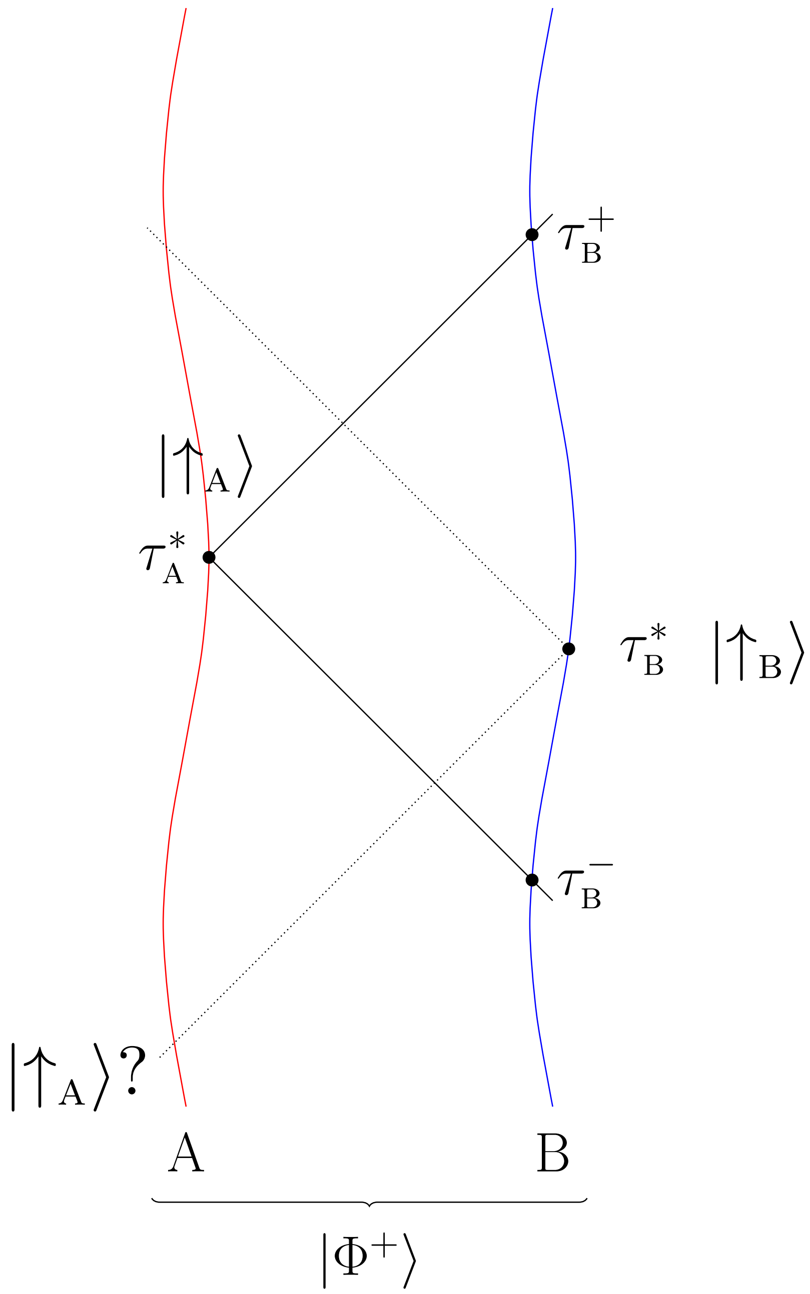

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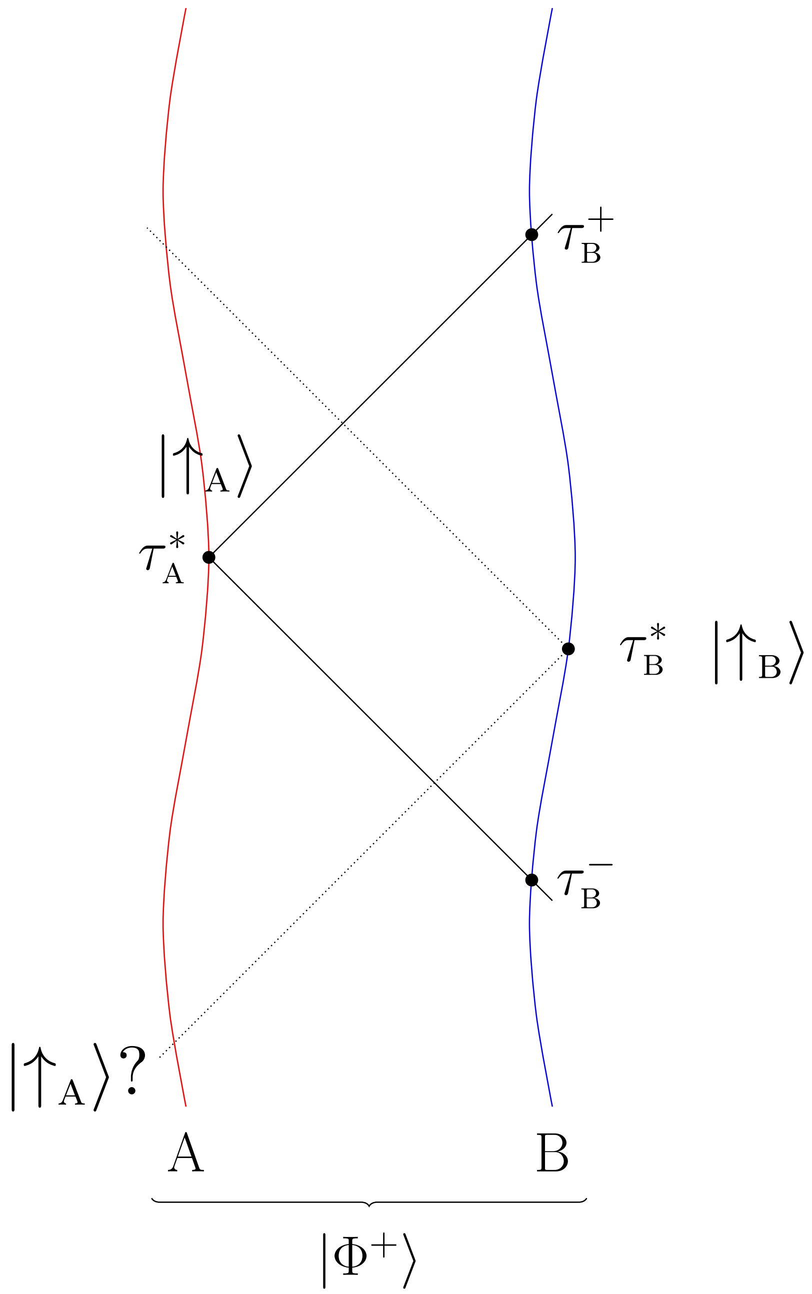

# Updates along the Future Lightcone?

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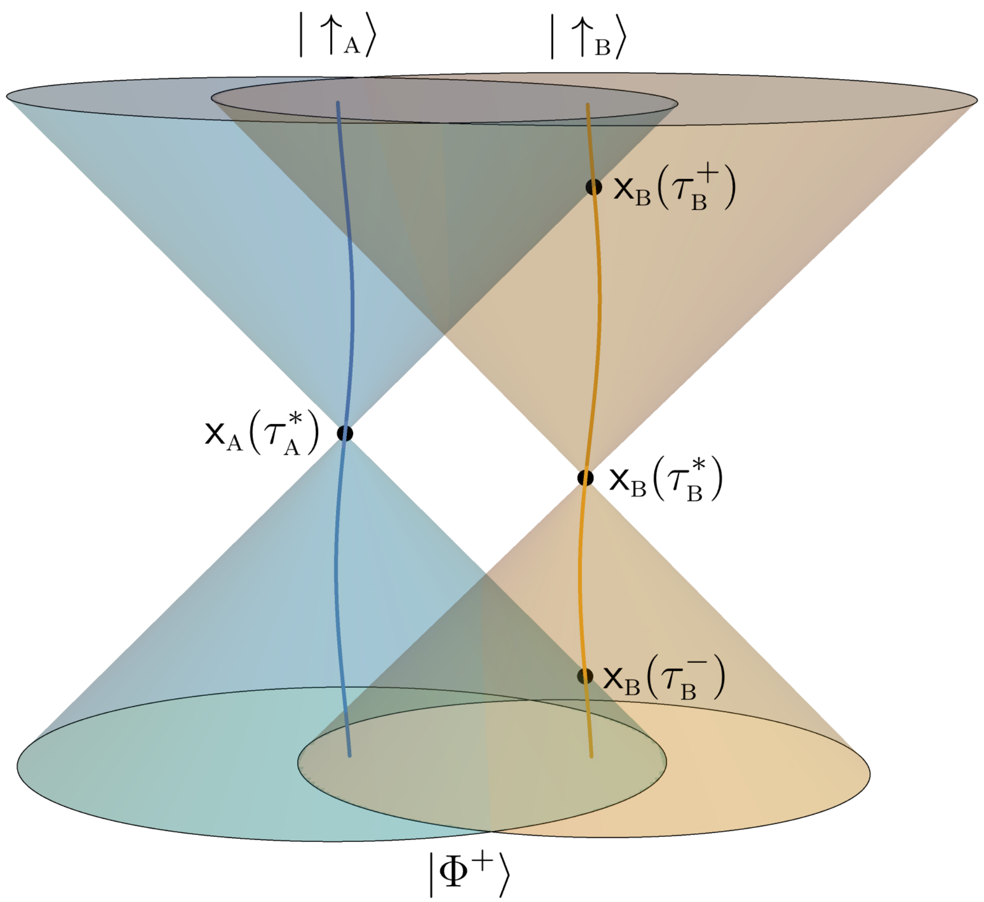


- In many ways an update along the future light cone is nice:

  -Suggestive of a notion of causal propagation of update

  -No retrocausation arguments

  -In a way it captures where information can propagate

Is it also problematic?

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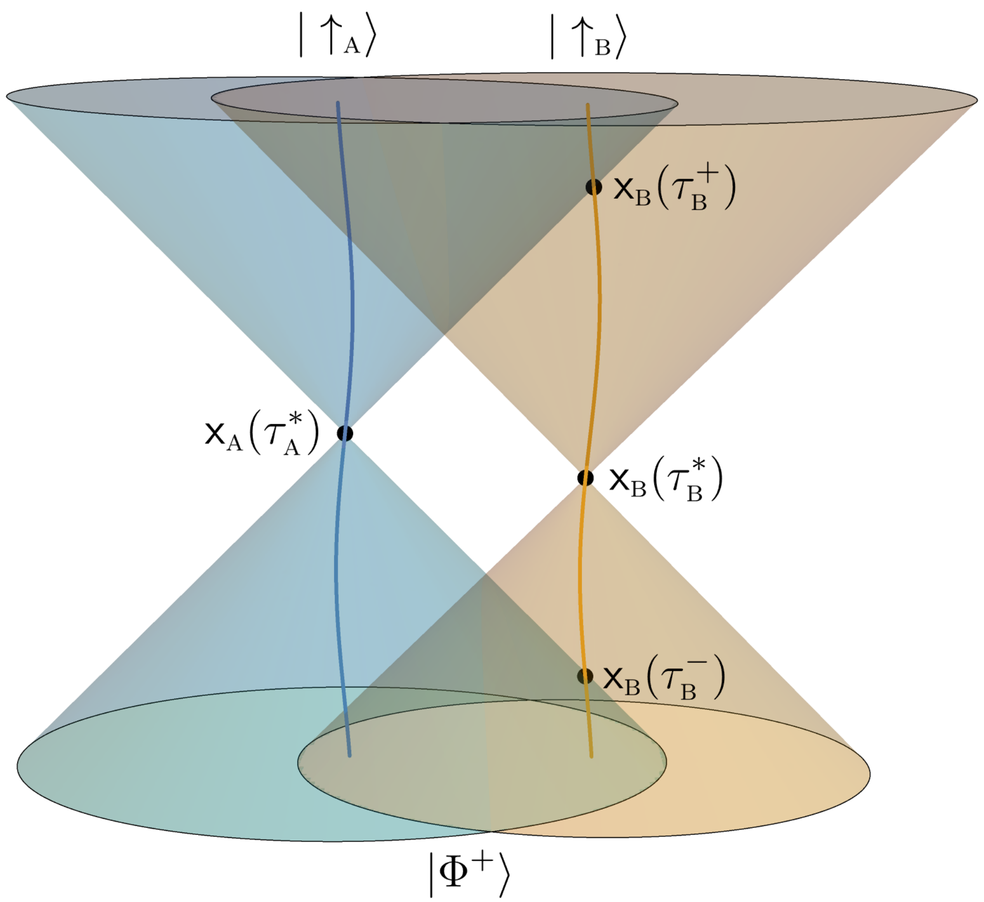


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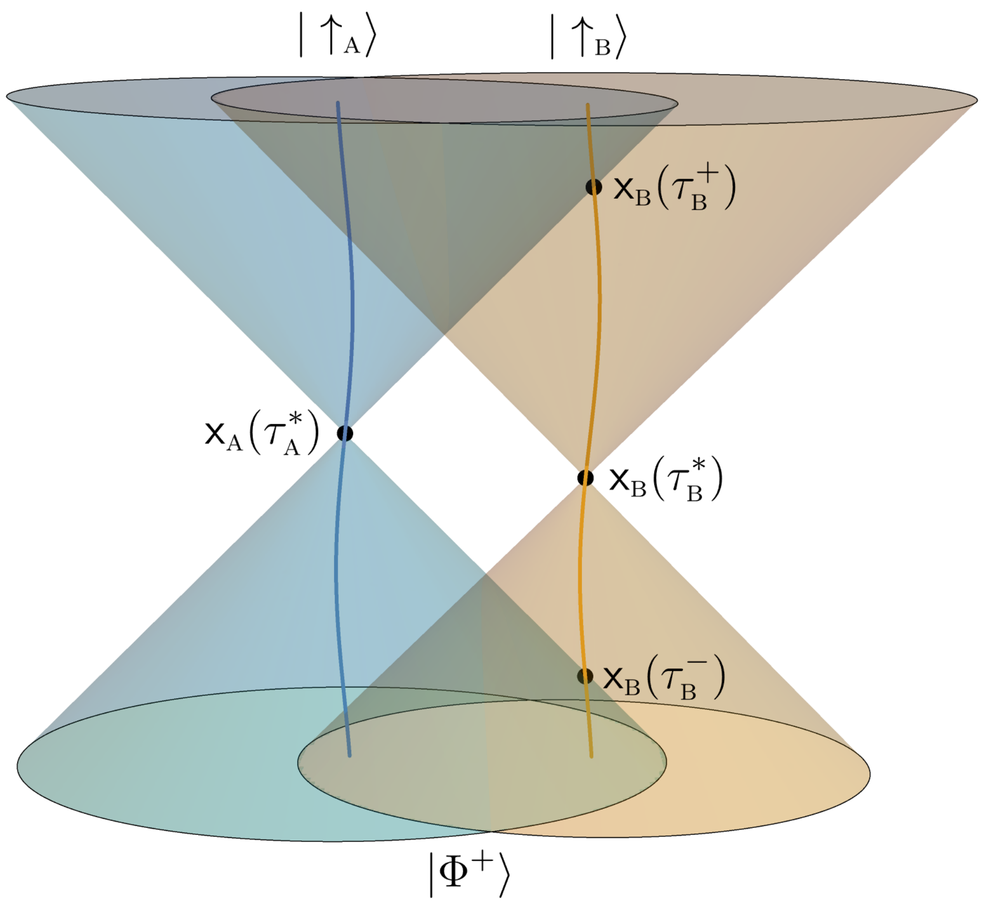
Is it also problematic? Yes!

# Updates along the Future Lightcone?

- Measuring a Bell pair $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|{\uparrow_A}\rangle\,|{\uparrow_B}\rangle + |{\downarrow_A}\rangle\,|{\downarrow_B}\rangle) \in \mathcal{H}_A \otimes \mathcal{H}_B$




Consider the expectation values of $\hat{\sigma}_{z,A}$ at some $\tau_A^{**} > \tau_A^*$, and of $\hat{\sigma}_{z,B}$ at a point spacelike separated from $x_A(\tau_A^*)$, at some $\tau_B^* \in (\tau_B^-, \tau_B^+)$. Then, an update along the future lightcone of the measurement on A predicts the following expectation values:

$$\langle\hat{\sigma}_{z,A}\rangle = 1 \quad \text{at} \quad x_A(\tau_A^{**}), \quad \langle\hat{\sigma}_{z,B}\rangle = 0 \quad \text{at} \quad x_B(\tau_B^*),$$

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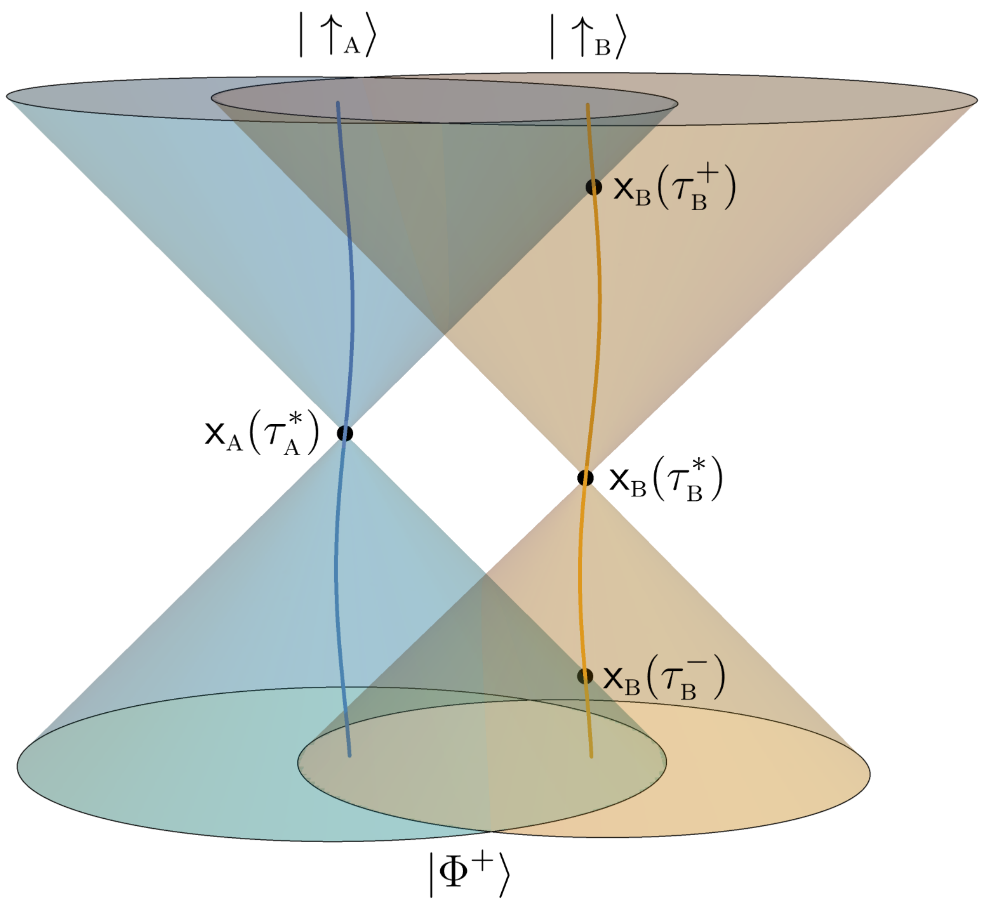


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and yet, because A and B are initially correlated, it must hold that

$$\langle\hat{\sigma}_{z,A} \otimes \hat{\sigma}_{z,B}\rangle = 1.$$

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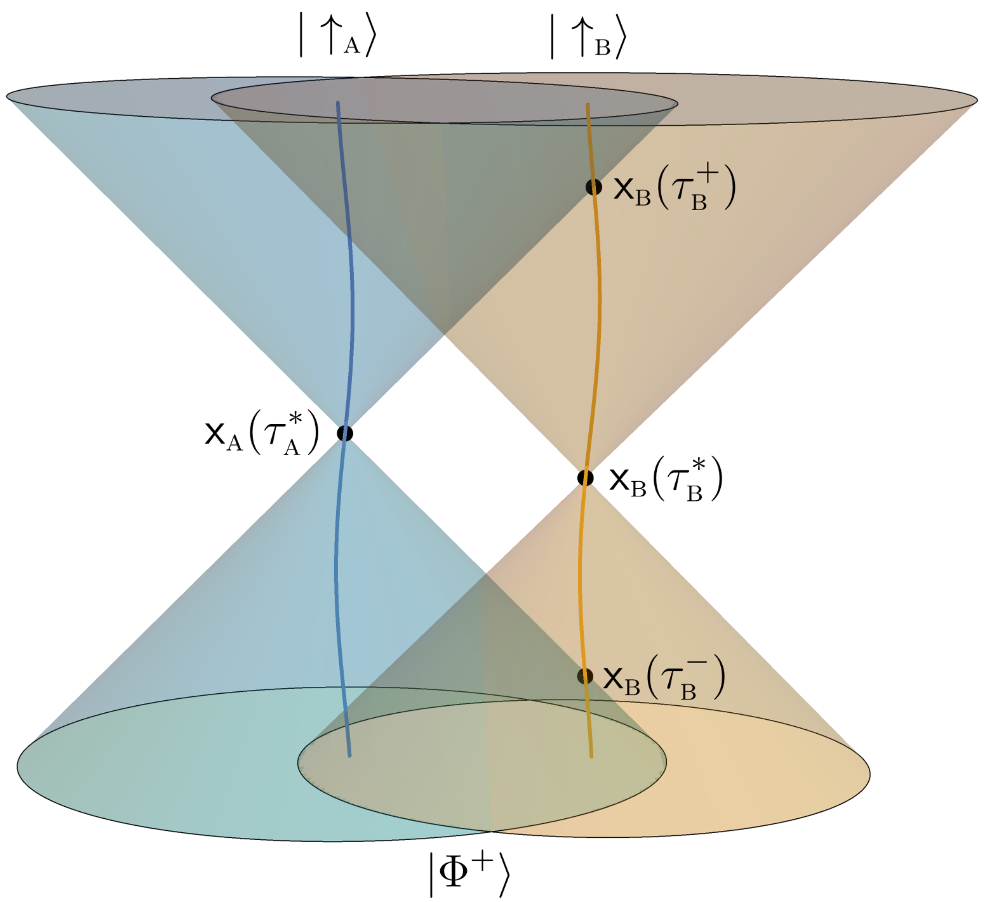


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No density operator $\hat{\rho}_{AB} \in \mathcal{L}(\mathcal{H}_A \otimes \mathcal{H}_B)$ can simultaneously satisfy both predicitons

# How to update then?

- A representation of multipartite system in terms of density operators does not leave room for a covariant update rule that is:

  $(i)$ is **fully predictive**, i.e., captures the outcome, correlations, and sequential measurements applied to both the joint system and its subsystems,

  $(ii)$ **respects ignorance**, i.e., information from measurements becomes available only through the causal structure.

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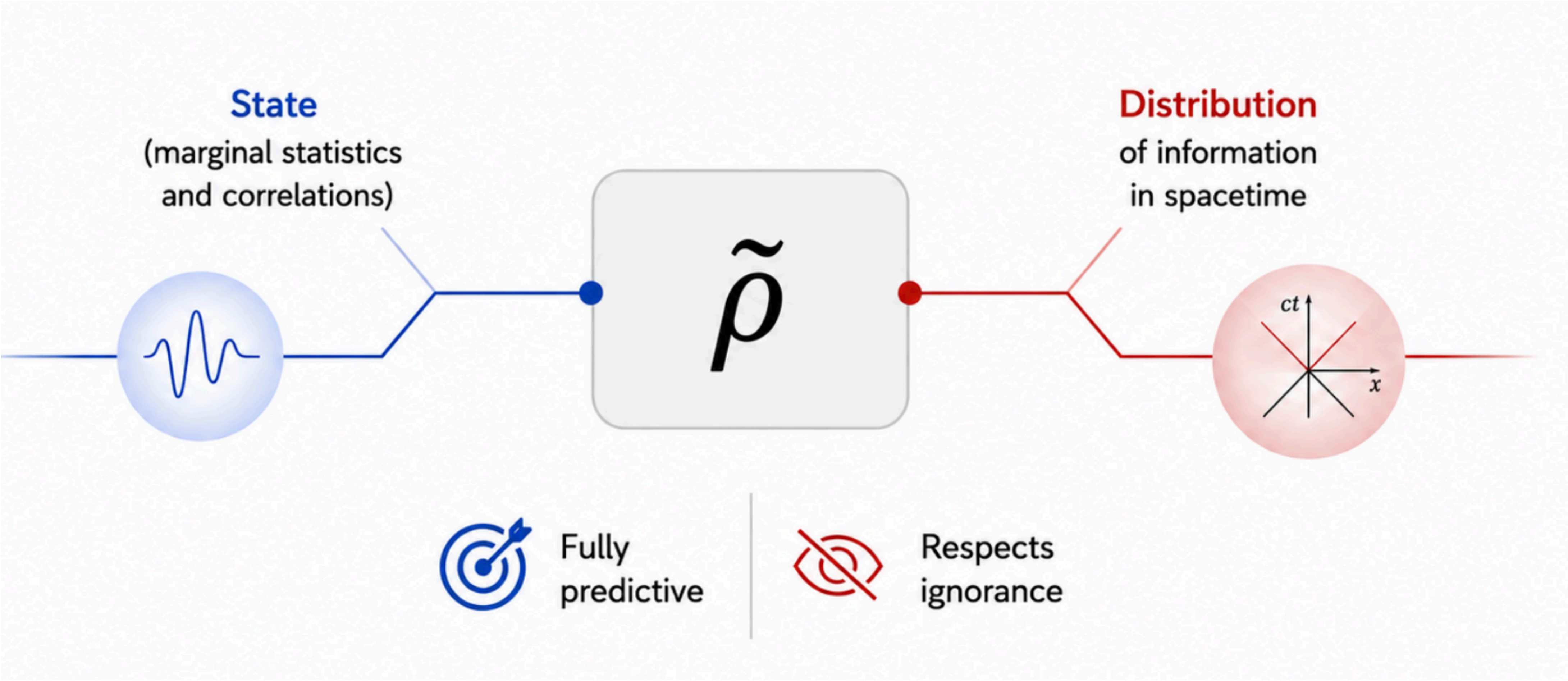
- Both are subject to the Aharonov–Albert charge-conservation argument

# How to update then?

- What is the minimal object that represents both the quantum state and the distribution of information in spacetime, while remaining fully predictive and respecting ignorance?



# Polyperspective formalism

- To resolve this tension and prescribe a state update that is fully predictive and causal, we need to introduce a formal distinction between:

  - *Individual observables* those that can be measured independently on A or B, and their expectations can be computed using *local* states $\hat{\rho}_{\mathrm{A}}$ and $\hat{\rho}_{\mathrm{B}}$
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- Implementing this distinction forces us to consider that the two following operators are different:

  - $\hat{A} \in \mathcal{L}(\mathcal{H}_{\mathrm{A}})$: An *individual* observable
  - $\hat{A} \otimes \mathbb{1}_{\mathrm{B}} \in \mathcal{L}(\mathcal{H}_{\mathrm{A}} \otimes \mathcal{H}_{\mathrm{B}})$ as a *joint* observable

Measuring an observable of A while having access to information from B is different from measuring the same observable without knowledge about B.

# Polyperspective formalism

- A convenient way of implementing this distinction is to consider the extended Hilbert space

$$\tilde{\mathcal{H}}_{\mathrm{AB}} \coloneqq \mathcal{H}_{\mathrm{A}} \oplus \mathcal{H}_{\mathrm{B}} \oplus (\mathcal{H}_{\mathrm{A}} \otimes \mathcal{H}_{\mathrm{B}})$$

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$$\tilde{\mathcal{H}}_{\mathrm{AB}} \coloneqq \mathcal{H}_{\mathrm{A}} \oplus \mathcal{H}_{\mathrm{B}} \oplus (\mathcal{H}_{\mathrm{A}} \otimes \mathcal{H}_{\mathrm{B}})$$

- And the polyperspective state $\tilde{\rho}_{\mathrm{AB}} \coloneqq \hat{\rho}_{\mathrm{A}} \oplus \hat{\rho}_{\mathrm{B}} \oplus \hat{\rho}_{\mathrm{AB}}$

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- How do we determine the poly(perspective) state?

# Polyperspective formalism

Consider the completely positive map $\Psi_{\mathcal{S}} : \mathcal{L}(\mathcal{H}_{\mathrm{A}} \otimes \mathcal{H}_{\mathrm{B}}) \to \mathcal{L}(\mathcal{H}_{\mathrm{A}} \otimes \mathcal{H}_{\mathrm{B}})$ encoding all the transformations undergone by A and B in $\mathcal{S} \subset \mathcal{M}$

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Set an initial state of the system $\hat{\rho}_{\mathrm{AB}}$

The local components of the polystate are

$$\hat{\rho}_{\mathrm{A}}(\tau_{\mathrm{A}}) \propto \mathrm{Tr}_{\mathrm{B}}\, \Psi_{\mathcal{J}^-(\mathsf{x}_{\mathrm{A}}(\tau_{\mathrm{A}}))}(\hat{\rho}_{\mathrm{AB}}), \qquad \hat{\rho}_{\mathrm{B}}(\tau_{\mathrm{B}}) \propto \mathrm{Tr}_{\mathrm{A}}\, \Psi_{\mathcal{J}^-(\mathsf{x}_{\mathrm{B}}(\tau_{\mathrm{B}}))}(\hat{\rho}_{\mathrm{AB}}),$$

where $\mathcal{J}^-(\mathsf{x})$ denotes the causal past of $\mathsf{x} \in \mathcal{M}$

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where $\mathcal{J}^-(\mathsf{x})$ denotes the causal past of $\mathsf{x} \in \mathcal{M}$

The joint state assigned to A and B should account only for transformations within *both* of their causal pasts, this prescribes:

$$\hat{\rho}_{\mathrm{AB}}(\tau_{\mathrm{A}}, \tau_{\mathrm{B}}) \propto \Psi_{\mathcal{J}^-(\mathsf{x}_{\mathrm{A}}(\tau_{\mathrm{A}})) \cup \mathcal{J}^-(\mathsf{x}_{\mathrm{B}}(\tau_{\mathrm{B}}))}(\hat{\rho}_{\mathrm{AB}}).$$

# Polyperspective formalism

$$\hat{\rho}_{\rm A}(\tau_{\rm A}) \propto {\rm Tr}_{\rm B}\, \Psi_{\mathcal{J}^-(\mathsf{x}_{\rm A}(\tau_{\rm A}))}(\hat{\rho}_{\rm AB}),$$

$$\hat{\rho}_{\rm B}(\tau_{\rm B}) \propto {\rm Tr}_{\rm A}\, \Psi_{\mathcal{J}^-(\mathsf{x}_{\rm B}(\tau_{\rm B}))}(\hat{\rho}_{\rm AB}),$$

$$\hat{\rho}_{\rm AB}(\tau_{\rm A}, \tau_{\rm B}) \propto \Psi_{\mathcal{J}^-(\mathsf{x}_{\rm A}(\tau_{\rm A})) \cup \mathcal{J}^-(\mathsf{x}_{\rm B}(\tau_{\rm B}))}(\hat{\rho}_{\rm AB}).$$

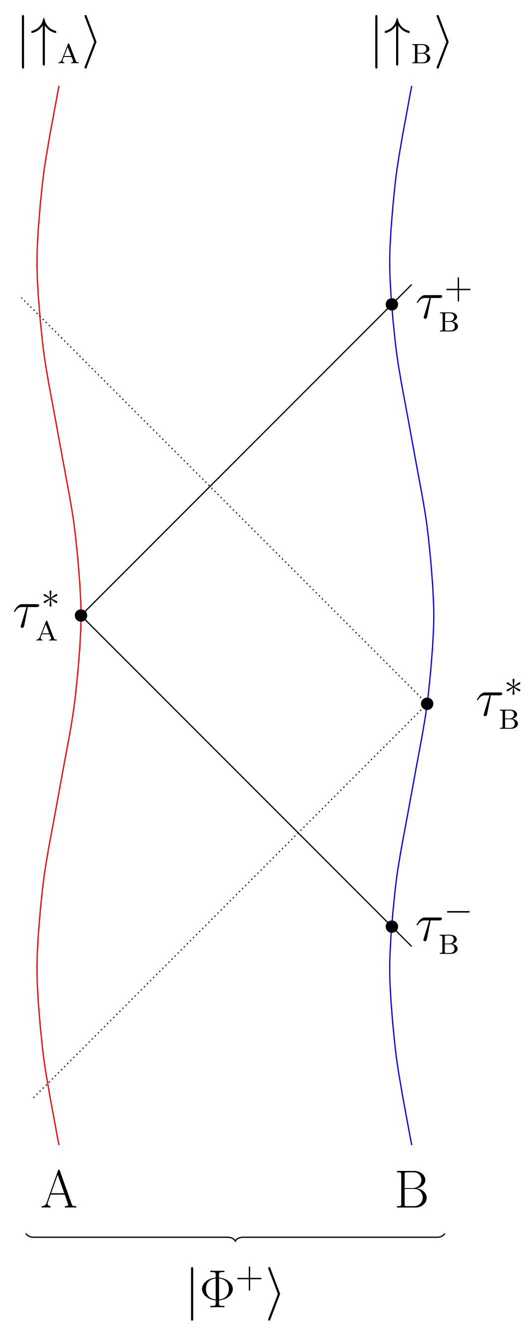
$$\tilde{\rho}(\tau_{\rm A}, \tau_{\rm B}) = \hat{\rho}_{\rm A}(\tau_{\rm A}) \oplus \hat{\rho}_{\rm B}(\tau_{\rm B}) \oplus \hat{\rho}_{\rm AB}(\tau_{\rm A}, \tau_{\rm B})$$

Information about Local observables

Information about Non-local observables

# Two foliations one polystate

Alice measures $\hat{\sigma}_z$ at $\tau_{\mathrm{A}} = \tau_{\mathrm{A}}^*$, and Bob measures $\hat{\sigma}_x$ at $\tau_{\mathrm{B}} = \tau_{\mathrm{B}}^*$, such that $\mathsf{x}_{\mathrm{A}}^* \equiv \mathsf{x}_{\mathrm{A}}(\tau_{\mathrm{A}}^*)$ is spacelike separated from $\mathsf{x}_{\mathrm{B}}^* \equiv \mathsf{x}_{\mathrm{B}}(\tau_{\mathrm{B}}^*)$. Both measure $+1$




$$\Psi_{\mathcal{S}}(\hat{\rho}) = \begin{cases} \hat{\rho} & \text{if } \mathsf{x}_{\mathrm{A}}(\tau_{\mathrm{A}}^*), \mathsf{x}_{\mathrm{B}}(\tau_{\mathrm{B}}^*) \notin \mathcal{S}, \\ |\uparrow_{\mathrm{A}}\rangle\langle\uparrow_{\mathrm{A}}|\, \hat{\rho}\, |\uparrow_{\mathrm{A}}\rangle\langle\uparrow_{\mathrm{A}}| & \text{if } \mathsf{x}_{\mathrm{A}}(\tau_{\mathrm{A}}^*) \in \mathcal{S}, \mathsf{x}_{\mathrm{B}}(\tau_{\mathrm{B}}^*) \notin \mathcal{S}, \\ |+_{\mathrm{B}}\rangle\langle+_{\mathrm{B}}|\, \hat{\rho}\, |+_{\mathrm{B}}\rangle\langle+_{\mathrm{B}}| & \text{if } \mathsf{x}_{\mathrm{A}}(\tau_{\mathrm{A}}^*) \notin \mathcal{S}, \mathsf{x}_{\mathrm{B}}(\tau_{\mathrm{B}}^*) \in \mathcal{S}, \\ |\uparrow_{\mathrm{A}}\rangle|+_{\mathrm{B}}\rangle\langle\uparrow_{\mathrm{A}}|\langle+_{\mathrm{B}}|\, \hat{\rho}\, |\uparrow_{\mathrm{A}}\rangle|+_{\mathrm{B}}\rangle\langle\uparrow_{\mathrm{A}}|\langle+_{\mathrm{B}}| & \text{if } \mathsf{x}_{\mathrm{A}}(\tau_{\mathrm{A}}^*), \mathsf{x}_{+}(\tau_{\mathrm{B}}^*) \in \mathcal{S}. \end{cases}$$

$$\tilde{\rho}_{ab}(\tau_{\mathrm{A}}, \tau_{\mathrm{B}}) = \begin{cases} \frac{1}{2}\mathbb{1}_{\mathrm{A}} \oplus \frac{1}{2}\mathbb{1}_{\mathrm{B}} \oplus |\Phi^+\rangle\langle\Phi^+| & \text{if } \tau_{\mathrm{A}} < \tau_{\mathrm{A}}^*, \tau_{\mathrm{B}} < \tau_{\mathrm{B}}^*, \\ |\uparrow_{\mathrm{A}}\rangle\langle\uparrow_{\mathrm{A}}| \oplus \frac{1}{2}\mathbb{1}_{\mathrm{B}} \oplus (|\uparrow_{\mathrm{A}}\rangle\langle\uparrow_{\mathrm{A}}| \otimes |\uparrow_{\mathrm{B}}\rangle\langle\uparrow_{\mathrm{B}}|) & \text{if } \tau_{\mathrm{A}} \geq \tau_{\mathrm{A}}^*, \tau_{\mathrm{B}} < \tau_{\mathrm{B}}^*, \\ \frac{1}{2}\mathbb{1}_{\mathrm{A}} \oplus |+_{\mathrm{B}}\rangle\langle+_{\mathrm{B}}| \oplus (|+_{\mathrm{A}}\rangle\langle+_{\mathrm{A}}| \otimes |+_{\mathrm{B}}\rangle\langle+_{\mathrm{B}}|) & \text{if } \tau_{\mathrm{A}} < \tau_{\mathrm{A}}^*, \tau_{\mathrm{B}} \geq \tau_{\mathrm{B}}^*, \\ |\uparrow_{\mathrm{A}}\rangle\langle\uparrow_{\mathrm{A}}| \oplus |+_{\mathrm{B}}\rangle\langle+_{\mathrm{B}}| \oplus (|\uparrow_{\mathrm{A}}\rangle\langle\uparrow_{\mathrm{A}}| \otimes |+_{\mathrm{B}}\rangle\langle+_{\mathrm{B}}|) & \text{if } \tau_{\mathrm{A}} \geq \tau_{\mathrm{A}}^*, \tau_{\mathrm{B}} \geq \tau_{\mathrm{B}}^*. \end{cases}$$

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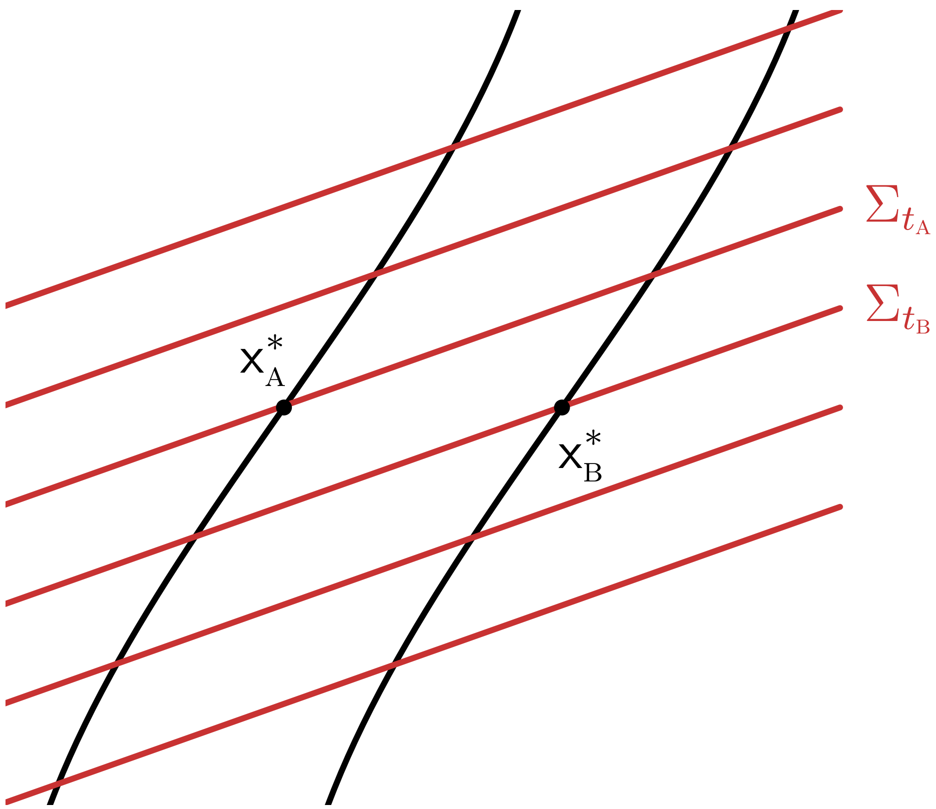


$$\hat{\rho}_\Sigma(t) = \begin{cases} |\Phi^+\rangle\langle\Phi^+| & \text{if } t < t_{\rm B} \\ |+_{\rm A}+_{\rm B}\rangle\langle+_{\rm A}+_{\rm B}| & \text{if } t_{\rm B} \leq t < t_{\rm A} \\ |\uparrow_{\rm A}+_{\rm B}\rangle\langle\uparrow_{\rm A}+_{\rm B}| & \text{if } t \geq t_{\rm A} \end{cases}$$

$$\tilde{\rho}_{ab}(\tau_{\rm A},\tau_{\rm B}) = \begin{cases} \frac{1}{2}\mathbb{1}_{\rm A} \oplus \frac{1}{2}\mathbb{1}_{\rm B} \oplus |\Phi^+\rangle\langle\Phi^+| & \text{if } \tau_{\rm A} < \tau^*_{\rm A}, \tau_{\rm B} < \tau^*_{\rm B}, \\ |\uparrow_{\rm A}\rangle\langle\uparrow_{\rm A}| \oplus \frac{1}{2}\mathbb{1}_{\rm B} \oplus (|\uparrow_{\rm A}\rangle\langle\uparrow_{\rm A}| \otimes |\uparrow_{\rm B}\rangle\langle\uparrow_{\rm B}|) & \text{if } \tau_{\rm A} \geq \tau^*_{\rm A}, \tau_{\rm B} < \tau^*_{\rm B}, \\ \frac{1}{2}\mathbb{1}_{\rm A} \oplus |+_{\rm B}\rangle\langle+_{\rm B}| \oplus (|+_{\rm A}\rangle\langle+_{\rm A}| \otimes |+_{\rm B}\rangle\langle+_{\rm B}|) & \text{if } \tau_{\rm A} < \tau^*_{\rm A}, \tau_{\rm B} \geq \tau^*_{\rm B}, \\ |\uparrow_{\rm A}\rangle\langle\uparrow_{\rm A}| \oplus |+_{\rm B}\rangle\langle+_{\rm B}| \oplus (|\uparrow_{\rm A}\rangle\langle\uparrow_{\rm A}| \otimes |+_{\rm B}\rangle\langle+_{\rm B}|) & \text{if } \tau_{\rm A} \geq \tau^*_{\rm A}, \tau_{\rm B} \geq \tau^*_{\rm B}. \end{cases}$$

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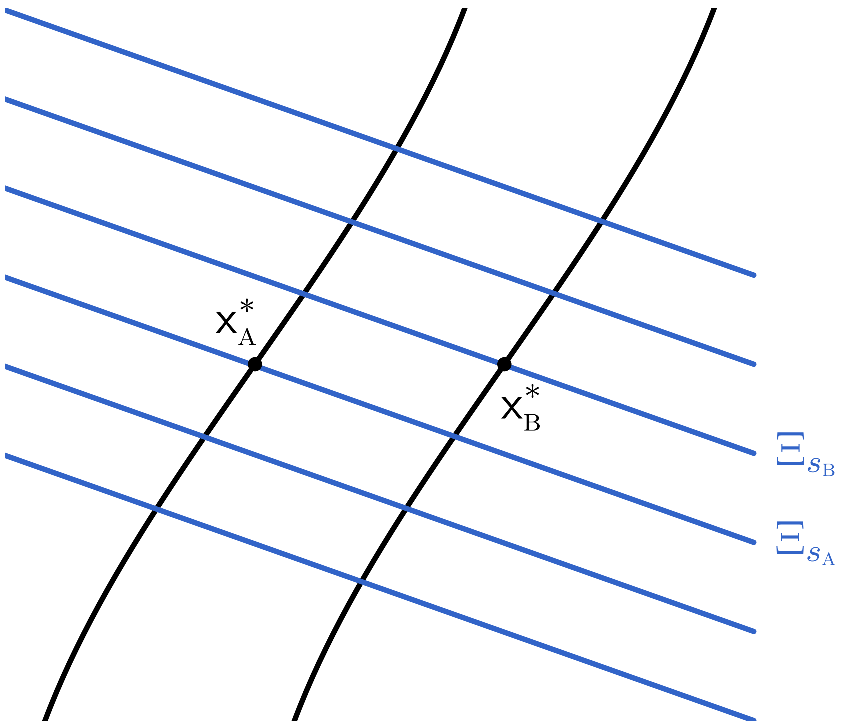
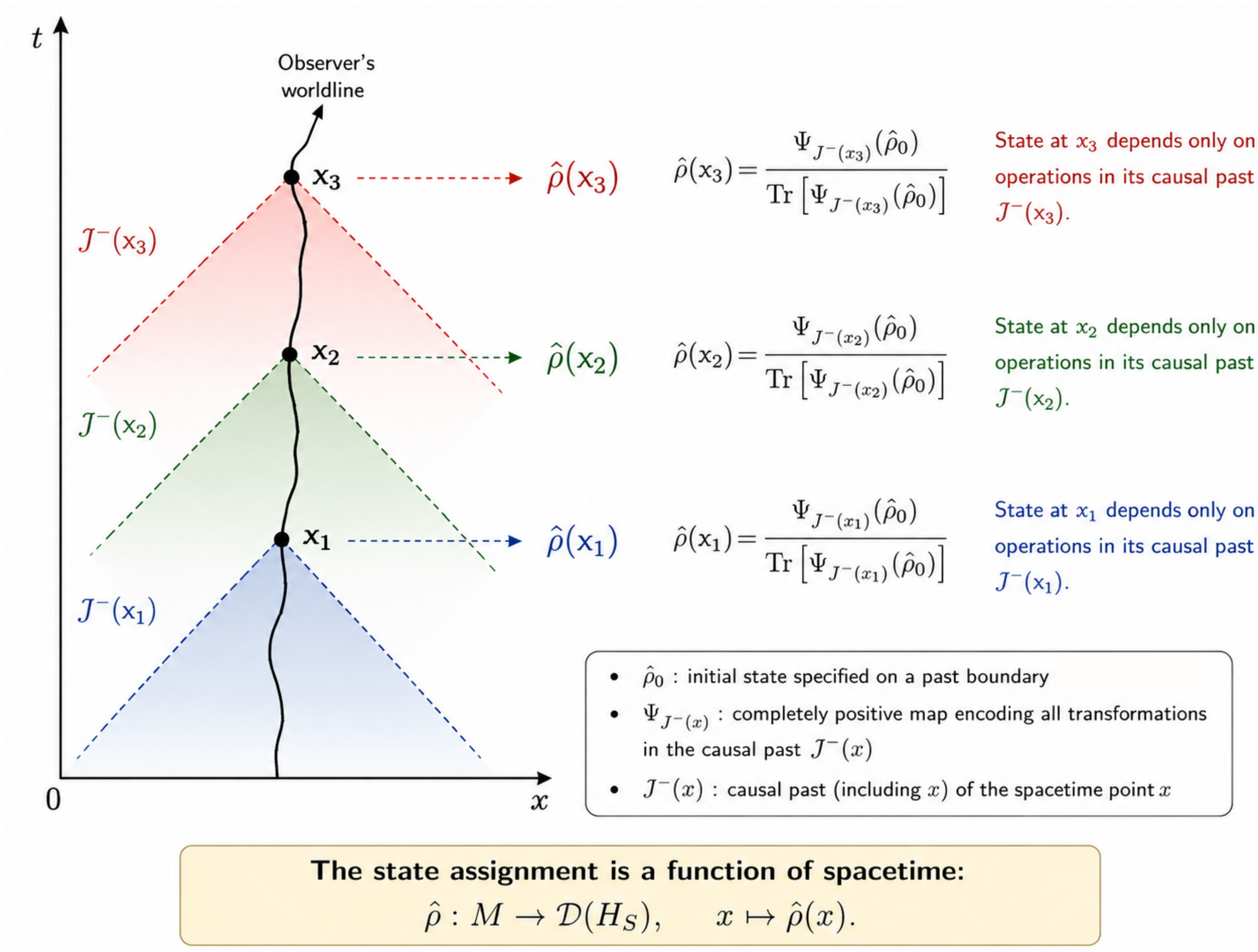



$$\hat{\rho}_{\Xi}(s) = \begin{cases} |\Phi^+\rangle\langle\Phi^+| & \text{if } s < s_{\text{A}} \\ |\uparrow_{\text{A}}\uparrow_{\text{B}}\rangle\langle\uparrow_{\text{A}}\uparrow_{\text{B}}| & \text{if } s_{\text{A}} \leq s < s_{\text{B}} \\ |\uparrow_{\text{A}} +_{\text{B}}\rangle\langle\uparrow_{\text{A}} +_{\text{B}}| & \text{if } s \geq s_{\text{B}} \end{cases}$$

$$\tilde{\rho}_{ab}(\tau_{\text{A}}, \tau_{\text{B}}) = \begin{cases} \frac{1}{2}\mathbb{1}_{\text{A}} \oplus \frac{1}{2}\mathbb{1}_{\text{B}} \oplus |\Phi^+\rangle\langle\Phi^+| & \text{if } \tau_{\text{A}} < \tau^*_{\text{A}}, \tau_{\text{B}} < \tau^*_{\text{B}}, \\ |\uparrow_{\text{A}}\rangle\langle\uparrow_{\text{A}}| \oplus \frac{1}{2}\mathbb{1}_{\text{B}} \oplus (|\uparrow_{\text{A}}\rangle\langle\uparrow_{\text{A}}| \otimes |\uparrow_{\text{B}}\rangle\langle\uparrow_{\text{B}}|) & \text{if } \tau_{\text{A}} \geq \tau^*_{\text{A}}, \tau_{\text{B}} < \tau^*_{\text{B}}, \\ \frac{1}{2}\mathbb{1}_{\text{A}} \oplus |+_{\text{B}}\rangle\langle+_{\text{B}}| \oplus (|+_{\text{A}}\rangle\langle+_{\text{A}}| \otimes |+_{\text{B}}\rangle\langle+_{\text{B}}|) & \text{if } \tau_{\text{A}} < \tau^*_{\text{A}}, \tau_{\text{B}} \geq \tau^*_{\text{B}}, \\ |\uparrow_{\text{A}}\rangle\langle\uparrow_{\text{A}}| \oplus |+_{\text{B}}\rangle\langle+_{\text{B}}| \oplus (|\uparrow_{\text{A}}\rangle\langle\uparrow_{\text{A}}| \otimes |+_{\text{B}}\rangle\langle+_{\text{B}}|) & \text{if } \tau_{\text{A}} \geq \tau^*_{\text{A}}, \tau_{\text{B}} \geq \tau^*_{\text{B}}. \end{cases}$$

# Generalization to Recollections



# Polyperpesctive formalism and Aharonov

**Aharonov & Albert's concern:** A relativistic state-update prescription should not spoil conserved charges.

## Local vs non-local observables

$\hat{q}(\mathbf{x})$ — **local charge density**

$\hat{Q}(\Sigma_t) = \int_{\Sigma_t} \mathrm{d}\Sigma\, \hat{q}(\mathbf{x})$ — **global charge (non-local)**

## Polystate bookkeeping

$\rho_x$ — Local predictions use the local sectors, e.g. $\langle \hat{q}(\mathbf{x}) \rangle_{\rho_x}$

$\rho_{\Sigma_t}$ — Global predictions use the joint sector, e.g. $\left\langle \int_{\Sigma_t} \mathrm{d}\Sigma\, \hat{q}(\mathbf{x}) \right\rangle_{\rho_{\Sigma_t}}$

## Global charge is computed from the joint sector

$$\left\langle \int_{\Sigma_t} \mathrm{d}\Sigma\, \hat{q}(\mathbf{x}) \right\rangle_{\rho_{\Sigma_t}} = Q \quad \neq \quad \int_{\Sigma_t} \mathrm{d}\Sigma\, \langle \hat{q}(\mathbf{x}) \rangle_{\rho_x}$$

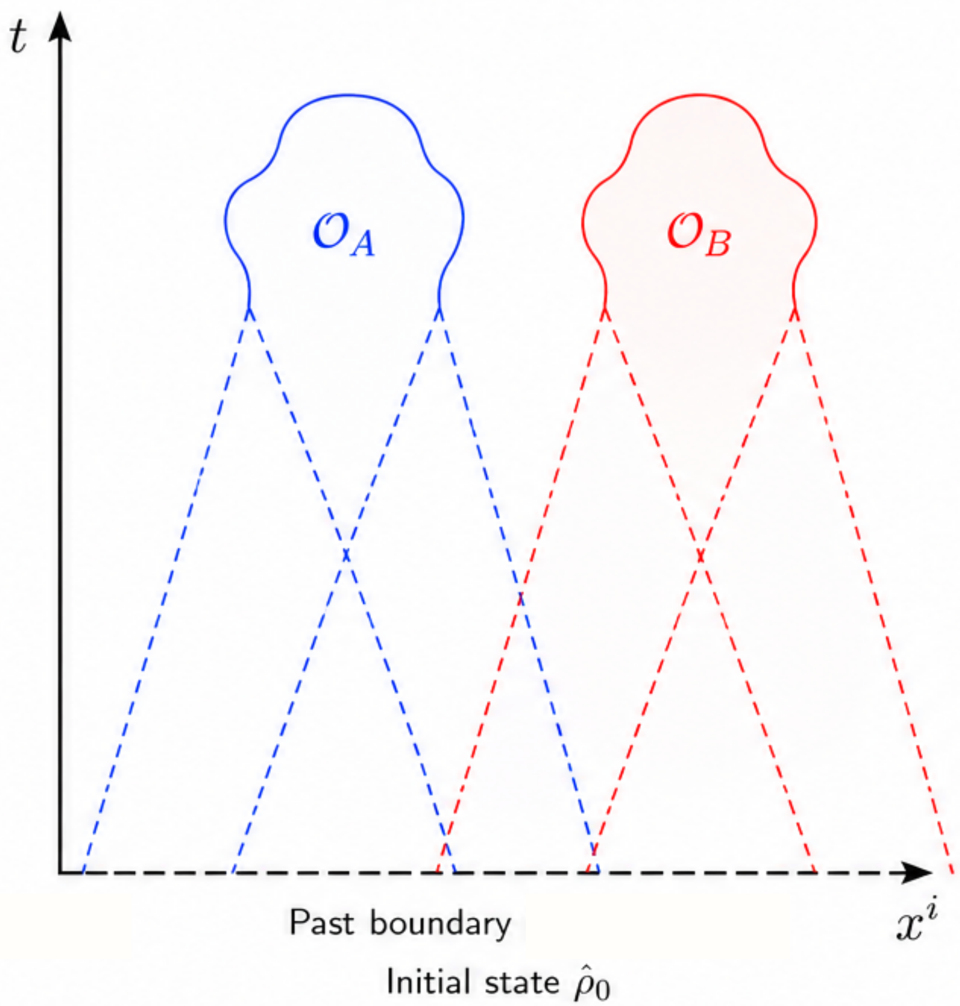
Expectation of the total charge (a non-local observable) computed using the **joint sector**.

Sum of local expectations computed in different **local sectors**. In general, not equal to the global result.

- ✔ Local and non-local observables are evaluated in **different sectors** of the polystate.
- ✔ The equality on the left is the **correct prediction** for the non-local observable $\hat{Q}(\Sigma_t)$.
- ✔ Therefore, **charge conservation is preserved** in the polystate framework.

# Localizable systems

- One can replace finite subsystems by spacetime regions. Heuristically:




### Region states

For any spacetime region $\mathcal{O} \subset \mathcal{M}$ (open, with suitable boundary), define the region state

$$\hat{\rho}_{\mathcal{O}} = \frac{\Psi_{J^-(\mathcal{O})}(\hat{\rho}_0)}{\mathrm{Tr}\left[\Psi_{J^-(\mathcal{O})}(\hat{\rho}_0)\right]}$$

where $\Psi_{J^-(\mathcal{O})}$ is the completely positive map encoding all dynamics in the causal past $J^-(\mathcal{O})$.

| | |
|---|---|
| $\hat{\rho}_{\mathcal{O}_A}$ | Information accessible to observers in region $\mathcal{O}_A$ (local observables in $\mathcal{O}_A$). |
| $\hat{\rho}_{\mathcal{O}_B}$ | Information accessible to observers in region $\mathcal{O}_B$ (local observables in $\mathcal{O}_B$). |
| $\hat{\rho}_{\mathcal{O}_A \cup \mathcal{O}_B}$ | Information required to predict joint observables across $\mathcal{O}_A$ and $\mathcal{O}_B$ (correlations). |

### Polystate for regions

The polystate is the direct sum of region states:

$$\tilde{\rho}_{AB} = \hat{\rho}_{\mathcal{O}_A} \oplus \hat{\rho}_{\mathcal{O}_B} \oplus \hat{\rho}_{\mathcal{O}_A \cup \mathcal{O}_B}$$

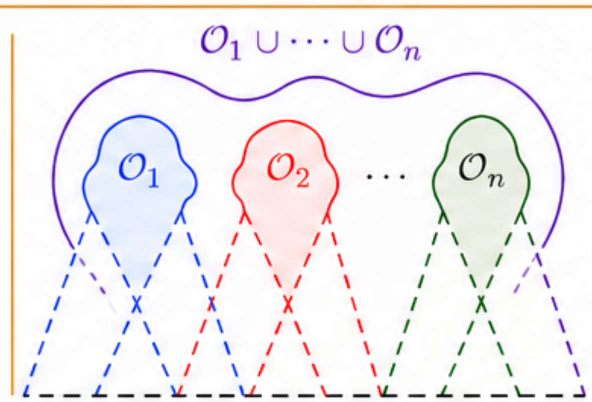
Local predictions use $\hat{\rho}_{\mathcal{O}_A}$ or $\hat{\rho}_{\mathcal{O}_B}$.
Non-local predictions use $\hat{\rho}_{\mathcal{O}_A \cup \mathcal{O}_B}$.

### Generalization to an arbitrary number of regions

For regions $\mathcal{O}_1, \ldots, \mathcal{O}_n$, assign states using only their causal pasts:

$$\hat{\rho}_{\mathcal{O}_I} = \frac{\Psi_{J^-(\mathcal{O}_I)}(\hat{\rho}_0)}{\mathrm{Tr}\left[\Psi_{J^-(\mathcal{O}_I)}(\hat{\rho}_0)\right]}, \qquad \mathcal{O}_I = \bigcup_{i \in I} \mathcal{O}_i$$

for any non-empty subset $I \subseteq \{1, \ldots, n\}$.

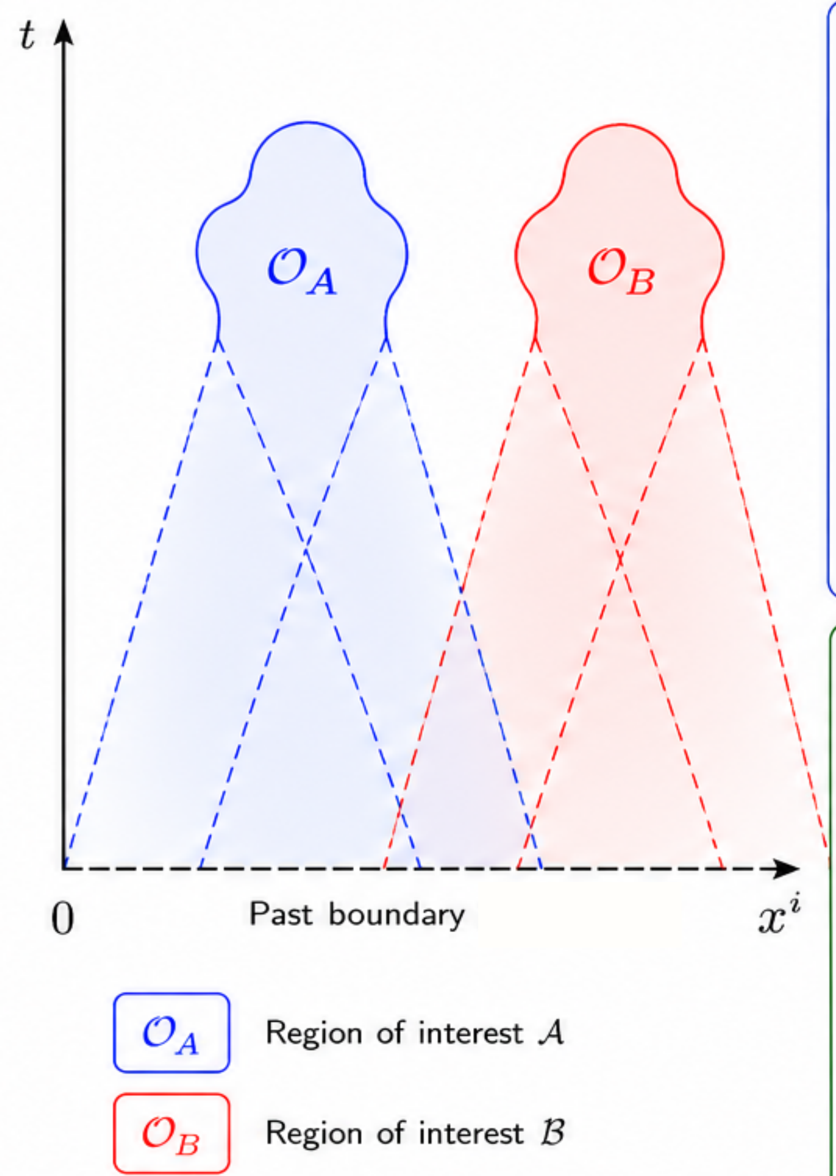



Polystate for the collection:

$$\tilde{\rho}_{1\cdots n} = \bigoplus_{\emptyset \neq I \subseteq \{1,\ldots,n\}} \hat{\rho}_{\mathcal{O}_I}$$

All predictions (local and non-local) follow from this decomposition.

# Bonus: AQFT extension




- $J^-(S)$: causal past of a set $S \subset \mathcal{M}$.

## 1. State assignments to regions

Let $\omega$ be a global state on the algebra $\mathcal{A}(\mathcal{M})$ of observables on spacetime $\mathcal{M}$.

For any region $\mathcal{O} \subset \mathcal{M}$, define a **(global)** state $\omega_{\mathcal{O}}$ by

$$\omega_{\mathcal{O}}(A) = \frac{\omega\left(\Psi^{\dagger}_{J^-(\mathcal{O})}(A)\right)}{\omega\left(\Psi^{\dagger}_{J^-(\mathcal{O})}(1)\right)}, \quad \forall A \in \mathcal{A}(\mathcal{M}).$$

Here $\Psi_{J^-(\mathcal{O})}$ is a completely positive map encoding all operations in the causal past $J^-(\mathcal{O})$.

$\omega_{\mathcal{O}}$ is **the global state associated with the information available to** $\mathcal{O}$.

In practice, it is used to compute observables of interest in $\mathcal{O}$ (or in unions containing $\mathcal{O}$).

## 2. Analogue of the polystate (two regions)

For two regions, the complete specification of predictions for local and joint observables is the collection of states

$$\mathcal{P}_{AB} = \{\omega_{\mathcal{O}_A}, \omega_{\mathcal{O}_B}, \omega_{\mathcal{O}_A \cup \mathcal{O}_B}\}.$$

Interpretation:

- $\omega_{\mathcal{O}_A}$ : all local predictions (expectations) in $\mathcal{O}_A$.
- $\omega_{\mathcal{O}_B}$ : all local predictions (expectations) in $\mathcal{O}_B$.
- $\omega_{\mathcal{O}_A \cup \mathcal{O}_B}$ : joint predictions and correlations across $\mathcal{O}_A$ and $\mathcal{O}_B$.

Local predictions use $\omega_{\mathcal{O}_A}$ or $\omega_{\mathcal{O}_B}$ ; non-local (joint) predictions use $\omega_{\mathcal{O}_A \cup \mathcal{O}_B}$.
This is a family of compatible state assignments, not a state on a single algebra.

## 3. Generalization to many regions

For a collection of regions $\{\mathcal{O}_1, \ldots, \mathcal{O}_n\}$, define for any non-empty subset $I \subseteq \{1, \ldots, n\}$ the union $\mathcal{O}_I = \bigcup_{i \in I} \mathcal{O}_i$. The polyperspective specification is the family

$$\mathcal{P}_{1\ldots n} = \{\omega_{\mathcal{O}_I}\}_{\emptyset \neq I \subseteq \{1,\ldots,n\}}.$$

All predictions for observables in any region $\mathcal{O}_I$ follow from $\omega_{\mathcal{O}_I}$.

# Summary

The **polyperspective formalism** extends standard quantum states to include subsystem-specific information. It provides the **minimal extension** of the density-operator formalism required to consistently represent different informational situations, differentiating clearly between individual and joint measurements.

**Fully predictive and respects ignorance**: Incorporates measurement outcomes and correct statistics while respecting the causal propagation of information.

**Covariance and conservation**: Respects covariance and resolves prior issues (e.g., charge conservation, Aharonov–Albert objection).

**Natural framework for detector-based measurement theory**: Provides a natural framework for relativistic quantum information protocols and measurement theory.

**Extends to quantum field theory**: The same construction admits a natural formulation in QFT through spacetime-dependent completely positive maps.

The problem is not solved by choosing a better update surface; it is solved by recognizing that **a single density operator is insufficient** to represent all relevant informational situations.

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