Relativistic Quantum Information
COST Action 23115

What happens between in and out? Events do.

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Summary

- QFT beyond S-matrix
- Events
- The limitations of von Neumann measurements
- Decoherent histories as a measurement theory
- The challenge of causality

QFT is more than S-matrix

- In textbooks, QFT is often presented as an S-matrix theory.
- High-energy experiments proceed through scattering. Modelled by preparation of an *in* state ($t \to -\infty$) and measurement w.r.t. an *out* state ($t \to +\infty$).
- However: "Although the S-matrix is a central concern of particle physics, it is not the only thing worth calculating. We sometimes need to calculate the expectation value of a Heisenberg-picture operator $O_H(t)$ in a state $|\alpha\rangle$ that is defined by its appearance at very early times. (This is the problem that particularly concerns us in calculating correlation functions in cosmology...)" (Weinberg, 2014).

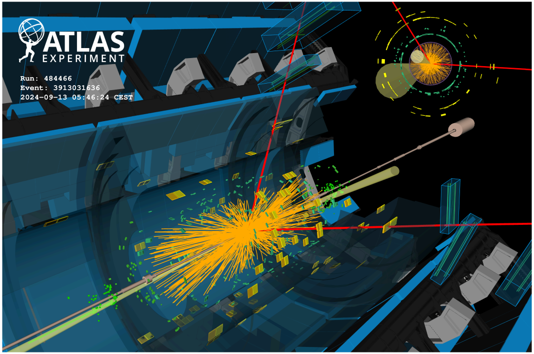
QFT is more than S-matrix

Physical predictions of QFT that cannot be captured by S-matrix.

- *Quantum optics:* photon bunching, anti-bunching, time-resolved interference.
- *Non-equilibrium QFT:* Early Universe cosmology. Temporal correlations in electron transport or ultra-cold gases.
- *Quantum information protocols:* photon teleportation.
- *Temporal measurements:* time of arrival, non-exponential decays.

All these phenomena happen between in and out, i.e., they involve measurements in real time.

What do we see between in and out?

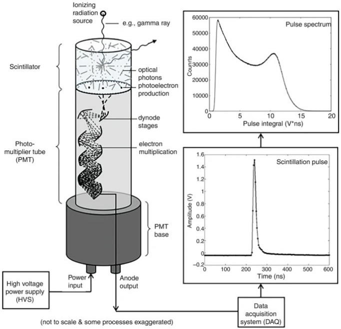


Records obtained through multiple channels: charged-particle hits, deposited energy in calorimeters, muon spectrometers.

A high-energy event may consist of thousands of elementary detection events.

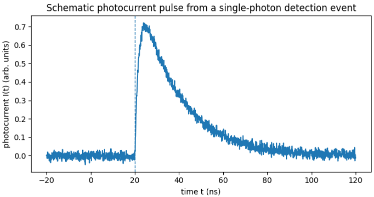
What do we see between in and out?

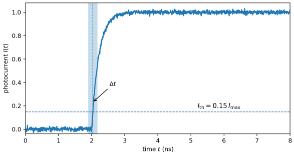
Measurement statistics are built out of elementary detection events



What do we see between in and out?

Individual detection events





The width of the detection pulse is due to the self-dynamics of the detector that eventually destroys the record.

The time of detection t_d corresponds to the photo-current $I(t)$ crossing a threshold value I_{th}.

First-crossing time is a **history observable**.

Properties of elementary detection events

- Events correspond to macroscopic records.
- They are associated with a spacetime region of small, but macroscopic, localization width.
- The spacetime point associated a detection event is typically a *random variable*, and not a control variable of the experiment.
- An elementary detection event may involve additional random variables, e.g., energy deposited in the detector.

How to describe events in QFT.

The first QFT measurement theory: *Glauber photo-detection*.

Hierarchy of joint probability densities of detection with respect to spacetime points x_i.

$$\begin{aligned} p_1(x) \\ p_2(x_1, x_2) \\ \dots \\ p_n(x_1, x_2, \dots, x_n) \end{aligned}$$

$$p_n(x_1, x_2, \dots, x_n) = C\langle\Psi|\hat{E}^{(-)}(x_1)\dots\hat{E}^{(-)}(x_n)\hat{E}^{(+)}(x_n)\dots\hat{E}^{(+)}(x_1)|\Psi\rangle$$

Our proposal: *Quantum Temporal Probabilities*

Use hierarchy of joint probability densities of detection with respect to spacetime points x_i.

$$p_n(x_1, q_1; x_2, q_2; \ldots ; x_n, q_n)$$

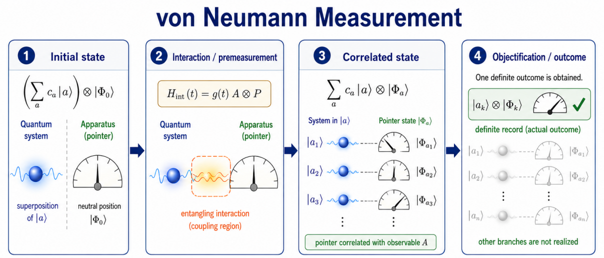
CA, B. L. Hu, and K. Savvidou, Ann. Phys. 450, 169239 (2023).

q_i correspond to other observables
(energy, momentum, angular momentum, and so on).

The probability density p_n is a linear functional of the $2n$-point correlation function

$$G^{(n,n)}(x_1, \ldots, x_n; x'_1, \ldots, x'_n) := Tr\left\{\mathcal{T}\left[\hat{O}(x_n)\ldots\hat{O}(x_1)\right]\right.$$
$$\left.\times\hat{\rho}_0\bar{\mathcal{T}}\left[\hat{O}(x'_1)\ldots\hat{O}(x'_n)\right]\right\}.$$

von Neumann measurements



Straightforwardly generalizes to n measurement events, each apparatus represented by a different Hilbert space.

Features of von Neumann measurements

1. The time of (pre)measurement is a control variable, not a random variable.
2. The time of objectification is not specified. It can be delayed indefinitely, even taken to infinity as in S-matrix theory.
3. Generalized to QFT with the spacetime region being the "control variable".

AQFT: C. J. Fewster and R. Verch, Comm. Math. Phys. 378, 851 (2020).

Detectors: J. Polo-Gómez, L. J. Garay, and E. Martín-Martínez, Phys. Rev. D 105, 065003 (2022); T. R. Perche, J. Polo-Gómez, B. de S. L. Torres, and E. Martín-Martínez, Phys. Rev. D109, 045013 (2024).

Limitation of von Neumann measurements

To paraphrase Wigner:

1. Von Neumann measurements answer the question: **Where is the particle now?**
2. A complete theory of measurements must also answer the question: **When is the particle here?**

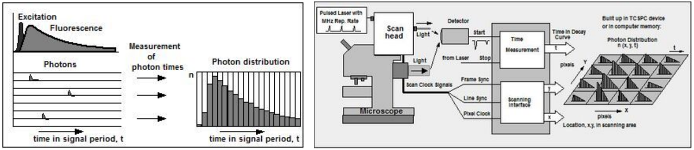


Figure: Time (or spacetime point) as a random variable in time-correlated single photon counting (TCSPC).

E.P. Wigner, in "Aspects of Quantum Theory", ed. by A. Salam, E.P. Wigner (Cambridge University Press, 1972)

Decoherent histories approach to measurement

1. Measurement corresponds to the emergence of a definite macroscopic record.
2. Such records correspond to decoherent histories for coarse-grained apparatus observables.
3. Can define events for which the time of occurrence is the random variable.

Decoherent histories: GellMann-Hartle-Isham axioms

Space $\mathcal{U}$ of possible histories: α, β, γ, etc.

1. Equipped with partial ordering $\leq$ for coarse-graining.
2. Zero (0) and unit (I) histories: $0 \leq \alpha \leq I$, for all $\alpha \in \mathcal{U}$.
3. Equipped with partial ordering $\prec$ for temporal succession. (+ axioms about causal structure)
4. Equipped with logical operation AND.

Examples

- Time ordered sequences of projectors: $\alpha := (\hat{\alpha}_{t_1}, \hat{\alpha}_{t_2}, \ldots, \hat{\alpha}_{t_n})$
- Feynman paths on configuration space, $\gamma : \mathbb{R} \to Q$.

M. Gell-Mann and J. B. Hartle, in 'Complexity, Entropy, and the Physics of Information', ed. by W. Zurek, (Addison Wesley, 1990); C. J. Isham, Quantum Logic and the Histories Approach to Quantum Theory, J. Math. Phys. 35, 2157 (1994).

Decoherent histories: GellMann-Hartle-Isham axioms

Space $\mathcal{UP}$ of history propositions: α, β, γ, etc.

1. Closure of $\mathcal{U}$ w.r.t. logical operations AND, OR, and NOT.
2. $\mathcal{UP}$ inherits both partial orderings from $\mathcal{U}$.

Example

Each discrete-time history $\alpha := (\hat{\alpha}_{t_1}, \hat{\alpha}_{t_2}, \ldots, \hat{\alpha}_{t_n})$ is represented by the tensor product operator

$$\tilde{\alpha} := \alpha_{t_1} \otimes \alpha_{t_2} \otimes \cdots \otimes \alpha_{t_n},$$

acting on the tensor-product Hilbert space

$$\mathcal{V} = \mathcal{H}_{t_1} \otimes \mathcal{H}_{t_2} \otimes \cdots \otimes \mathcal{H}_{t_n}.$$

Then $\mathcal{UP}$ is the set of all projectors on $\mathcal{V}$.

This is the *history projection operator* (HPO) representation.

Decoherent histories: GellMann-Hartle-Isham axioms

Decoherence functional: $d : \mathcal{UP} \times \mathcal{UP} \to \mathbb{C}$.

1. $d(I, I) = I$; $d(0, \alpha) = 0$.
2. Hermitian: $d(\alpha, \beta) = d^*(\beta, \alpha)$.
3. Normalized: $d(\alpha \text{ OR } \beta, \gamma) = d(\alpha, \gamma) + d(\beta, \gamma)$, if α AND $\beta = 0$.

Example

For a discrete-time history $\alpha := (\widehat{\alpha}_{t_1}, \widehat{\alpha}_{t_2}, \ldots, \widehat{\alpha}_{t_n})$, define $\hat{C}_\alpha = \hat{\alpha}_{t_n}(t_n) \ldots \hat{\alpha}_{t_1}(t_1)$ in terms of the Heisenberg-picture projectors $\hat{\alpha}_{t_i}(t_i) = e^{i\hat{H}t_i} \hat{\alpha}_{t_i} e^{-i\hat{H}t_i}$. Then,

$$d(\alpha, \beta) = Tr\left(\hat{C}_\alpha \hat{\rho}_0 \hat{C}_\beta^\dagger\right).$$

Decoherent histories approach to measurements

Let α_i define an exhaustive and exclusive set of histories for a microscopic system + macroscopic apparatus:

- each α_i is highly coarse-grained, and defines a distinct macroscopic record on an apparatus.
- $|d(\alpha_i, \alpha_j)| < \epsilon |d(\alpha_i, \alpha_i|$ for $\epsilon << 1$.

Then Prob(α_i) is the probability for the associated measurement outcome.

With an appropriate selection of histories, we can describe measurements in which time is a random variable.

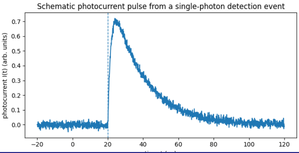
Time of an event as a random variable

Let $\mathcal{H} = \mathcal{H}_+ \oplus \mathcal{H}_-$ with associated projectors $\hat{P}$ and $\hat{Q}$.
The probability that the transition occurs at time t in an interval B is

$$\mathrm{Prob}(t \in B) = \int_B dt \int_B dt' \langle \psi_0 | \hat{C}_{t'}^\dagger \hat{C}_t | \psi_0 \rangle,$$

where $\hat{C}_t := e^{i\hat{H}t} \hat{P} \hat{H} \hat{S}_t$. Here, $\hat{S}_t$ is the restricted propagator on $\mathcal{H}_-$.

For measurements, $\mathcal{H}_+$ corresponds to the signal being above the threshold.



CA and K. Savvidou, PRA86, 012111 (2012).

QTP hierarchy

$$P_n(x_1, q_1; x_2, q_2; \ldots; x_n, q_n) = \int d^4y_1 \ldots d^4y_n R_{(1)}(y_1, q_1) \ldots R_{(n)}(y_n, q_n)$$
$$\times G^{(n,n)}(x_1 - \frac{1}{2}y_1, \ldots, x_n - \frac{1}{2}y_n; x_1 + \frac{1}{2}y_1, \ldots, x_n + \frac{1}{2}y_n)$$

where $R_{ab}(y, q)$ is the *detection kernel*—containing all information about detector.

Schematically,

$$P_n(z_1, z_2, \ldots, z_n) = G^{\alpha_1\alpha_2\ldots\alpha_n}_{\beta_1\beta_2\ldots\beta_n} \; {}^{(1)}R^{\beta_1}_{\alpha_1}(z_1) \; {}^{(2)}R^{\beta_2}_{\alpha_2}(z_2) \ldots \; {}^{(n)}R^{\beta_n}_{\alpha_n}(z_n).$$

This equation defines a non-normalized POVM

$$P_n(z_1, z_2, \ldots, z_n) = Tr[\hat{\rho}_0 \hat{E}_{z_1,\ldots z_n}].$$

QTP hierarchy

The QTP hierarchy (like Glauber's hierarchy) follows from the leading-order terms w.r.t. field-detector coupling. Its derivation requires no switching on and off of an interaction.

Applications: Photodetection beyond Glauber, relativistic time of arrival, time-correlations in Unruh/Hawking radiation, particle oscillation formulas, relativistic q-info measures.

Interestingly, the QTP result in this approximation can be reproduced by a variation of von Neumann measurements, assuming that objectification time is randomly distributed during the interaction period.

Key question

Are the field correlation functions the essential physical content of a QFT?

They mostly are: in S-matrix, in the Schwinger-Keldysh formulation of QFT, in Glauber's theory, in QTP.

The problem with events

A series of results culminating to Hegerfeld's theorems:

Localization implies superluminal tails in the probabilities for subsequent outcomes.

Let $|C\rangle$ be a state obtained by an action, localized in a spacetime region C, on the vacuum $|\Omega\rangle$. Then

$$\langle C|\hat{E}_x|C\rangle \neq 0, \quad \text{for } \; x \; \text{ spacelike separated from } C$$

Reeh-Schlieder theorem suggests that $\langle \Omega|\hat{E}_x|\Omega\rangle \neq 0$, but we find

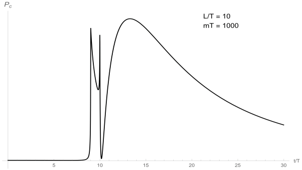
$$\langle C|\hat{E}_x|C\rangle > \langle \Omega|\hat{E}_x|\Omega\rangle.$$

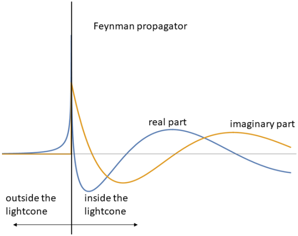
Hegerfeld tails dominate over Reeh-Schlieder dark counts.

G. C. Hegerfeldt, Phys. Rev. Lett. 72, 596. (1994); Annalen der Physik 7, 716 (1998).

The problem with events

Hegerfeld tails are a consequence of positive energy and unitarity.





Equivalently: fields propagate through retarded propagator, but amplitudes through Feynman propagator.

Hegerfeld tails cannot be argued away, unless one is willing to also throw away the notion of spacetime localized records.

Possible solution

In decoherent histories, probability densities are defined with an error ϵ, due to inexactness of decoherence condition.

Need a theorem:

The inherent uncertainty Δp in Reeh-Schlieder dark count probability makes it impossible to infer the action C from the Hegerfeld tails.

Without such a theorem, it is difficult to keep events and not do violence to the essential content of QFT. We are very far from a general proof.

Steps towards quantifying RS dark count: R. Falcone and C. Conti, Phys. Rev. D 113, L061702 (2026).

To discuss:

- There was good reason to restrict QFT to S-matrix observables, but this is not feasible any more.
- A general theory of QFT measurements must describe events—hence, include the treatment of the spacetime coordinates of macroscopic records as random variables.
- Do current QFT frameworks require extension to account for QFT measurements?
- A new axiomatization based on QFT correlations and associated operations as in Schwinger-Keldysh. Or perhaps, adapting the GHI axioms?

Thank you!